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COMPOSITE ESTIMATORS
IN THE JUNE HOG SERIES

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ABSTRACT

The National Agricultural Statistics Service (NASS) issues quarterly hog statistics for inventory items such as total hogs, breeding hogs, and market hogs. This study examined six composite estimators using historical data for June from 1979 to 1986. The purpose of this study was to evaluate the characteristics of the six composite estimators for use by the Agricultural Statistics Board (ASB), a committee of NASS experts, in setting official statistics for hog inventories.

The evaluation, which involved three sets of analyses, showed that of the six composite estimators the smoothed inverse variance composite most closely approximated historical ASB results.

KEYWORDS: Hog Series, Composite Estimation, Exponential Smoothing, Bias, Mean Square Error, Variance.

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SUMMARY

The National Agricultural Statistics Service (NASS) issues quarterly hog statistics for inventory items such as total hogs, breeding hogs, and market hogs (including several weight groups). This study examined six composite estimators having the following weighting schemes: equal, inverse variance, inverse coefficient of variation, midrange, smoothed inverse variance, and smoothed inverse coefficient of variation. The analyses used historical data for June from 1979 to 1986. The purpose of this study was to evaluate the characteristics of the six composite estimators for use by the Agricultural Statistics Board (ASB) in setting official statistics for hog inventories.

The study employed three primary methods:

- 1. Multivariate analyses of the biases -- treating the ASB estimates as truth,
- 2. Nonparametric analyses for four evaluation criteria -- bias, average absolute difference, standard deviation, and root mean square error -- treating the ASB estimates as truth, and,
- 3. Model interpretation analyses of ASB estimates in terms of the six composites.

The multivariate analyses of the biases showed that for most states those composites which depended heavily on the multiple frame estimator were less biased than the other composites. The nonparametric analyses strongly indicated that the smoothed inverse variance composite was the "best" composite when all four evaluation criteria were considered. The model interpretation analyses revealed that, with one exception, past ASB estimates most closely followed either the inverse variance or smoothed inverse variance composite (these estimators are similar in practice). In Iowa, the ASB models were closest to the mid-range models.

In summary, the smoothed inverse variance composite most closely approximated historical ASB results.

INTRODUCTION

The National Agricultural Statistics Service (NASS) of the U.S. Department of Agriculture (USDA) conducts quarterly agricultural surveys in March, June, September, and December. The June survey serves as the base for the survey cycle. Specifically, the tract (Y_1) , farm (Y_2) , weighted (Y_3) , and multiple frame (Y_4) estimates (indications) for items such as total hogs, breeding hogs, market hogs, market hogs under 60 pounds, etc. as listed in Table 1. These four indicators are all really multiple frame screening estimators. Each indication combines a list frame estimate and an area frame estimate. Greater detail about the indicators is available in Nealon (7).

The Agricultural Statistics Board (ASB), a committee of senior NASS statisticians from headquarters and major state statistical offices, uses the four survey indications and other non-survey information, such as hog slaughter and administrative data, to set official hog inventory estimates. These estimates reflect the expert judgment of the ASB members based on all available information. The four indications generally do not have the same numerical value because of both sampling and nonsampling errors. Generally, nonsampling errors cause the expected values of the four estimators to differ.

The objective of this study was to evaluate six composite estimators for the hog series using historical tract, farm, weighted, and multiple frame summary statistics from the June survey. Three considerations motivated this study:

- 1. Combining the four survey indicators statistically would contribute to better use of available information.
- 2. A composite estimator closer to the theoretically optimal composite estimator would have a variance less than or equal to any component indicator.
- 3. A method not influenced by changes in the membership of the ASB would make the estimation process more repeatable.

¹ The evaluations in this study used only June data because NASS reduced the scope of the December survey to the multiple frame states in 1987.

The study evaluated six composite estimators and the multiple frame indicator for eight hog series items within the context of the June survey. There were three major parts to this evaluation:

- 1. Multivariate analyses of the biases -- treating the ASB estimates as truth,
- 2. Nonparametric analyses for four evaluation criteria -- bias, average absolute difference, standard deviation, and root mean square error -- treating the ASB estimates as truth, and,
- 3. Model interpretation analyses of ASB estimates in terms of the six composites.

To improve readability, this report is divided into five sections and six appendices. The five sections provide a general description of the various analysis techniques employed and a summary of the conclusions that follow. The appendices contain the detailed supporting mathematical and statistical theory, technical definitions, statistical test procedures, and test results to support the analyses and conclusions of this report. The appendices also contain graphs of the various composite series, indication series and ASB series and tables of summary statistics for the weights in the four composites with variable weights.

The five sections are:

- 1. A description of composite estimation with definitions of the composites evaluated,
- A description of the data with the limitations which these data imposed on the analysis techniques,
- 3. Multivariate analyses of bias,
- 4. Nonparametric methods, and,
- 5. Model interpretations of the data.

The six appendices are:

- Composite estimators and their true variances,
- Graphs of indications, ASB estimates and composites,
- 3. Tables of summary statistics for the weights in variable weight composites,
- 4. Summary tables for multivariate analyses for biases,
- 5. Summary tables for nonparametric analyses for four criteria, and,
- 6. Summary tables of model interpretations of ASB estimates.

DESCRIPTION OF COMPOSITES

The composite estimators under investigation are all weighted averages of the NASS tract, farm, weighted, and multiple frame indicators used in the June survey. Thus, each composite has the following symbolical form:

$$Y_{C} = W_{1} * Y_{1} + W_{2} * Y_{2} + W_{3} * Y_{3} + W_{4} * Y_{4}$$

where the sum of the weights w_1 , w_2 , w_3 , and w_4 is one and Y_1 , Y_2 , Y_3 , and Y_4 represent the June tract, farm, weighted, and multiple frame indications, respectively, for a specified hog series item.

Both practical and theoretical considerations were the basis for choosing the six composites for evaluation. A combination of Agency requirements, available data, and the experience of other investigators were used in selecting the composites for evaluation. The theoretically optimal composite was not included in this evaluation because the historical data necessary to compute its weights were not available.²

The following list of composites contains, when appropriate, some theoretical justification and discussion of their origin. Both the numbers and short descriptive names will simplify reference.

1. Equal:

Each indication has an equal weight of 0.25.

2. Inverse Variance (Inv.var):
Each indication takes the associated inverse estimated variance as the weight.

² The formula for the weights in the minimal variance composite requires the covariance matrix of the component estimators. Similarly the formula for the weights in the minimal mean square error composite requires the mean square error matrix of the component estimators (see Appendix A). Estimates for these matrices are not available from the historical hog series.

- 3. Inverse CV (Inv.cv):
 Each indication takes the associated inverse estimated coefficient of variation as the weight.
- 4. Mid.range:
 The largest and smallest indications have a weight of one half while all other indications have a weight of zero.
- 5. Smoothed Inverse Variance (S.inv.var):
 Each indication takes the exponential smoothed
 historical average of the inv.var weights defined above
 as the associated weight³.
- 6. Smoothed Inverse CV (8.inv.CV):
 Each indication takes the exponential smoothed
 historical average of the inv.cv weights defined above
 as the associated weight.
- 7. Multiple Frame (Multi.frame):
 The multiple frame indication has a weight of one and all other indications have a weight of zero.

The equal and mid.range composites are both simple and easy to compute. The equal composite is the optimal composite when the individual component indicators are all unbiased, uncorrelated, and have equal variances.

The inv.var composite has the intuitively appealing property of giving large weights to indications with small variances and small weights to indications with large variances. The inv.var composite is the optimal composite when values used in computing the weights are the true variances, and, the individual component indications are unbiased and uncorrelated.

The inv.cv and mid.range composites are both ad hoc procedures and have no basis in theory. The inv.cv composite gives large weights to indications with small variances, as does the inv.var. The study included the inv.cv because exploratory analyses with a small data set suggested that the inv.cv followed closely the final ASB

³ The smoothed inverse variance weight for an indication in a given year was computed by taking 0.25 times the inverse variance weight in that year and adding 0.75 times the smoothed inverse variance weight from the previous year.

estimates for some items. The mid.range composite differs from the other composites by using only information about the level of the individual indications and not statistical reliability in the weighting formulas. Addition of the mid.range composite to the analyses occurred because the ASB estimates appeared to be approximately halfway between the largest and smallest indications for many items.

The s.inv.var and s.inv.cv composites employ smoothing for the following reasons:

- 1. Smoothing limits the variation in the composite caused by changes in the weights from survey to survey.
- 2. Composite weights should reflect the statistical reliability of the component indicators in the current survey. If the true variances of the indications are fairly stable over time, then smoothed weights may be more reliable than weights estimated with data from the current survey exclusively.

Notice that the smoothed weights adjust with time to reflect changes in the statistical reliability of the individual component indications.

The study added the multiple frame indicator to serve as a comparison and reference estimator. As the most reliable of the four indications in terms of variance, the multiple frame indicator is also the only indication available from all quarterly surveys.

The analyses that follow do not make use of the variance of the equal, inv.var, or s.inv.var composites. However, under certain conditions, which are spelled out in Appendix A, the large sample variance of these composites is given by:

$$Var(Y_C) = \underline{\mu}_W^T \underline{\Sigma}_Y \underline{\mu}_W + \underline{\mu}_Y^T \underline{\Sigma}_W \underline{\mu}_Y + tr(\underline{\Sigma}_W \underline{\Sigma}_Y)$$

where $\underline{\mu}_{W} = E(\underline{W})$ and $\underline{\mu}_{Y} = E(\underline{Y})$ and, where Σ_{Y} and Σ_{W} are the covariance matrices associated with the indicators and their weights, respectively. The first term in this formula is associated with the variance of the indications; the second term is associated with the variance of the weights; and, the third term is associated with the compounding of these two sources of variance in the composite.

DESCRIPTION OF DATA SETS

Two data sets were used in this study. One data set was used to calculate the six composites. The other data set was used to evaluate the composites. Composites were calculated for each of the eight hog items for which the ASB publishes June estimates.

The original intent was to evaluate the six composites for the ten largest hog producing states (Georgia, Illinois, Indiana, Iowa, Kansas, Minnesota, Missouri, Nebraska, North Carolina, and Ohio). Unfortunately, a review of historical data sets revealed that the weighted indicator was not available for Georgia in 1980 and for North Carolina in 1979. In addition, there were no variance estimates available before 1979 for market hogs greater then 180 pounds. Before 1979 this item consisted of two parts: hogs from 180 to 219 pounds and hogs above 220 pounds. This left a usable data set for eight states from 1979 to 1986.

Originally, the intent was to use both historical ASB data and historical balance sheet data (a reconciliation of statistical estimates with administrative records of hog movement and marketing) to evaluate the six composites. However, the balance sheet method proved impractical because marketing records were only available at the national level; there were no records for interstate movement. As a result, the historical ASB published estimates were the only data available for evaluating the performance of the six composites at the aggregate and state levels.

Table 1 summarizes the characteristics of the data sets used in this study. The various parts of Table 1 provide information about the indications used in the composites, the items examined in the study, and the data used in evaluating the composites. In summary, because of changes in the weight groups and sampling plans, usable June data was available only for eight states (Illinois, Indiana, Iowa, Kansas, Minnesota, Missouri, Nebraska, and Ohio) from 1979 to 1986.

TABLE 1 HOG AND PIG DATA SERIES UNDER STUDY

Ind	ications Data Set:	Abbreviation
	Tract Direct Expansion	Tract
2.	Farm Direct Expansion	Farm
3.	Weighted Direct Expansion	Weighted
4.	Multiple Frame Direct Expansion	Mult.Frame
 Bva	luation Data Set:	
1.	First State Recommendation	ASB 1
2.	First ASB Estimate	ASB 2
	First State Recommended Revision	ASB 3
	First ASB Revision	ASB 4
5.	Second State Recommended Revision	ASB 5
6.	Second ASB Revision	ASB 6
 Ite	ms In All Data Sets:	
1.	Total hogs	Total
2.	Market hogs	Market
	a. Less than 60 pounds	Under60
	b. 60 through 119 pounds	60-119
	c. 120 through 179 pounds	120-179
	d. 180 pounds and greater	180up
3	Hogs for Breeding	Breed
	Previous quarters pig crop	Births

ANALYSES

Graphical displays of the four indication series, the six composite series and the ASB series are presented in Appendix B. Four of the composites have weights that change over time. Summary statistics for these weights are displayed in Appendix C.

The following three sub-sections provide a description of the analyses. Each section uses a different technique to examine the relationship between the six composite estimates and historical ASB estimates. The conclusions from the three sets of analyses combine to support the final recommendations.

Multivariate Analyses of Biases

Multivariate techniques were used to analyze the biases of the composites in relation to the first ASB estimate, first ASB revision, and second ASB revision. Tables summarizing the eight-state aggregate analyses are given in the text. Tables and detailed commentary summarizing the state-bystate analyses are given in Appendix D.

Differences between the composites and the ASB estimates were used as the basic data in both the aggregate and state level analyses. The eight-state aggregate level analyses examined three ASB estimates, seven different composite estimators, and two methods of computing the eight-state estimate. In the state level analyses the two methods were replaced by the eight individual states.

The differences for each of the eight hog categories (total, breeding, market, under 60, 60-119, 120-179, 180 and up, and births) were the response variables in the multivariate model. The multivariate model symbolically had the following form:

$$Y_{ijkl} = \mu + s_i + B_j + c_k + s_{ij} + s_{ik} + s_{ijk} + s_{ijkl}$$

for $i=1,\ldots,8$; $j=1,\ldots,3$; $k=1,\ldots,7$; $l=1,\ldots,8$; and where S, B, and C represent the state, ASB, and composite main effects in the state analyses; and where S, B, and C represent methods of obtaining the eight-state estimate, ASB, and composite main effects in the eight-state aggregate

analyses. The second order interactions are denoted by SB, BC and SC and the third order interaction by SBC.

Analyses of the aggregate data determined those composites which differed significantly from ASB estimates due to composite, ASB, or method of aggregation effects. The two methods of obtaining an aggregate composite estimate were the following:

- 1. Computing the composite estimates for each individual state and summing the state results to get an aggregate composite estimate.
- 2. Computing the composite estimates directly from the aggregated indications.

When the weights have constant values, then both methods produce the same estimates.

The state analyses examined composite, ASB, and state effects on the ASB and composite differences. The analyses were multivariate because differences for each of the eight hog item categories (total, breed, market, under 60, 60-119, 120-179, 180up, and births) were analyzed simultaneously.

Comparison of the aggregate and state multivariate analyses shows that Iowa has more effect on the aggregate total than do the states with fewer hogs. Several states' averages were closest to the multiple frame or smoothed inverse variance composite. The aggregate totals and Iowa averages were both closest to the midrange composite for most categories.

Analyses of Aggregate Composites

The analyses of the eight-state aggregate data showed that the two methods of computing the eight-state aggregate composite were statistically different (method had a statistically significant effect on the level of the composite at the $\alpha=0.05$ significance level; Tukey's multiple comparisons test was employed in the sequel when necessary). The analyses also showed that the aggregate composites were statistically different. However, the initial ASB and two ASB revisions were not statistically different at $\alpha=0.05$.

The first method of computing an aggregate composite, summing the state composites, revealed statistically significant differences among composites for five of the

eight hog item categories; specifically, total, breed, market, under60, and 120-179. The second method of computing an aggregate composite, computing from the aggregated indications, had statistically different composites for four of the eight hog item categories; that is, total, breed, market, and under60. The midrange composite and the multiple frame showed the least total bias on average.

Tables 2, 3 and 4 below give further details to the previous commentary about the aggregate analyses. The first two tables compare the significantly different composites under the two methods of obtaining the aggregate totals. For example, Table 2 shows that for a significance level of 0.0001 and a Tukey's multiple comparison procedure at $\alpha = 0.05$ that the inv.var composite differs most from the mid.range (composite 4), then, the equal composite (composite 1), the s.inv.cv (composite 6), and, finally, the inv.cv (composite 3). The table only presents the one way ordering of the results so that each of the above mentioned composites are also different from the multiple frame individually (see Note 4 of Table 2).

Table 2 shows that many composites differ significantly from each other when the method of aggregation is that of summing the state composites to produce the aggregate composite estimates. Table 3 shows that only a few composites differ significantly when the method of aggregation is that of compositing the sum. Most differences occur for the multiple frame composite for the total, breeding, market, and under 60 categories. Comparing locations of the significant differences in Tables 2 and 3 shows that the six composites are more similar when the method of aggregation is that of compositing the sum of the state estimates than when the method of aggregation is that of summing the state composites.

Table 4 highlights the earlier statement that Iowa dominates the aggregate totals. The mid.range (composite 4) is often the composite which has an eight-year average that is nearest the ASB estimate at the eight-state level. Only the hog items of 60-119 and 120-179 are closest to the m.frame for both methods. However, the 180up item does show the equal composite closest for the second method. Since these comparisons do not come from statistical tests, no associated probabilities are available.

TABLE 2 SIGNIFICANTLY DIFFERENT COMPOSITES FOR AVERAGE BIAS FOR 1979 - 1986 METHOD 1: SUM OF COMPOSITES

_______ HOG AND PIG ITEM COMPOSITE TOTAL BREED MARKET U60 60-119 120-179 180UP BIRTH equal 4163 416 4163 41 inv.var inv.cv mid.range s.inv.var 4 41 s.inv.cv mult.frame 4163 4163 4163 4163 61 Pr > F.0001 .0001 .0001 .0001 .3609 .0028 .9067 .0742 Note 1: The composite numbering is defined in the Description of Composites, pages 4 and 5. The Pr>F is the level of significance in the ANOVA Note 2: for rejecting the hypothesis that all the composites are the same for that hog item. Note 3: The ordering of the composites gives the order in which the composites are most different from each other. Note 4: Only the one-way ordering of the differences are in the table. For example, the m.frame line shows that the m.frame is different from composites 4163 for the total. This means that the mult.frame (composite 7) is also significantly different for composites 1, 3, 4, and 6, respectively

TABLE 3 SIGNIFICANTLY DIFFERENT COMPOSITES FOR AVERAGE BIAS FOR 1979 - 1986 METHOD 2: COMPOSITE OF SUM

	HOG AND PIG ITEM
COMPOSITE	TOTAL BREED MARKET U60 60-119 120-179 180UP BIRT
inv.cv	4 4 4
mid.range s.inv.var s.inv.cv	4
mult.frame	e 416352 (Note 2) 4163
Pr >F	.0001 .0001 .0001 .0004 .5714 .0640 .9925 .173
~=======	
Note 1:	See notes 1 through 4 of Table 2.
Note 2:	The BREED and MARKET items have the same significance ordering (416352) as does the TOTAL item.

TABLE 4 COMPOSITE WITH LEAST AVERAGE BIAS (AVERAGED OVER EIGHT YEARS)

======	======			******			*****	=====
			HOG	AND PIG	ITEM			·
METHOD	TOTAL	BREED	MARKET	UNDER60	60-119	120-179	180UP	BIRTHS
1	4	4	4	4	7	7	4	4
2	4	4	4	4	7	7	1	4

Note 1	: See	note 1	l Table	2.				
=======	=====	-======	******		======	=======		

Analyses of Biases at the State Level

The state analyses showed that both type of composite and state significantly effected the differences between the composites and the ASB estimates. The effects varied form state to state. The multiple frame is statistically different at $\alpha=0.05$ from other composites for total, breed, market, under60, 60-119, and the 120-179 categories for all states except Minnesota and Missouri. No composite was statistically different at $\alpha=0.05$ for the 180up and the births' categories in any state (see Appendix D).

An examination of the average bias by states and by hog categories revealed that each of the estimators had a different number of least biased categories. Specifically, the estimators had the following number of least biased categories out of the total 64 (8 categories times 8 states) as follows: equal weights composite, five categories; the inverse variance, nine; the inverse cv, four; the midrange, 12; the smoothed inverse variance, six; the smoothed inverse cv, four; and, the multiple frame, 24. Iowa had the most categories (six) closet to the midrange average while Illinois had the most closet to the multiple frame (six).

Nonparametric Analyses for Four Criteria

Nonparametric (distribution-free) analyses of the composites examined both the state and aggregate level composites using four evaluation criteria; that is, bias, absolute difference, standard deviation and root mean square error. The state analyses focused on the effects of states for each item, the effects of items for each state, and the effects of both items and states. The aggregate analyses examined the effects of items for each of the two methods of computing an aggregate composite. The second ASB revised aggregate estimates were treated as truth in these analyses.

Giving equal weights to the four evaluation criteria for the nonparametric analyses strongly suggested that the smoothed inverse variance composite (s.inv.var) should be the composite of choice. However, there were differences between the results from the aggregate analyses and the state-by-state analyses. These differences were primarily a result of the large effect that Iowa had on the aggregate totals. Each state in the aggregate analyses contributed proportionately to the aggregate total, but the across-state analyses of the hog items permitted equal influence to each

state regardless of size. Table 5 (parts a and b) below shows that the s.inv.var is the "best" composite on the average for all hog items except the 120-179 item.

TABLE 5.A EQUAL TREATMENT OF ABSOLUTE BIAS, ABSOLUTE DIFFERENCE, ROOT MEAN SQUARE ERROR, AND STANDARD DEVIATION.

		HOG A	ND PIG I	PEM	
COMPOSITE	TOTAL	BREED	MARKET	UNDER60	60-119
equal	6.0	5.5	6.2	5.0	6.1
inv.var	3.1	3.2	2.9	4.0	2.8
inv.cv	3.7	3.4	3.8	3.4	3.9
mid.range	5.9	5.4	5.9	5.1	6.0
s.inv.var	2.3**	2.7**	2.1**	2.7**	2.4**
s.inv.cv	3.9	3.8	4.0	3.4	4.0
mult.frame	3.2	4.0	3.1	4.4	2.8

Note: The **'s indicate the that composite has the smallest average rank.

TABLE 5.B CATEGORIES CONTINUED.

		HOG AND	PIG ITEM	
COMPOSITE	120-179	180UP	BIRTH	AVERAGE
equal	6.2	5.0	4.8	5.6
inv.var	2.0**	4.5	3.8	3.3
inv.cv	4.0	4.3	3.3	3.7
mid.range	5.8	4.2	5.0	5.4
s.inv.var	2.6	3.1**	3.4**	2.7**
s.inv.cv	4.5	3.3	3.6	3.8
mult.frame	2.9	3.5	4.1	3.5

A description of the formulas for preparing the data for analysis and obtaining estimates follows. Let the actual population value be y and the value of the indication I be y_I . Then the mean square error (mse), bias, and mean absolute difference (absdif) are the following:

mse =
$$1/n * \Sigma(y_I - y)^2$$
 bias = $1/n * \Sigma(y_I - y)$
absdif = $1/n * \Sigma|y_T - y|$.

The relationship between the mse, bias, and variance (var) is the following:

$$var = mse - bias^2$$
.

The formulas for the root mean square error (rmse) and the standard deviation (std) are the following:

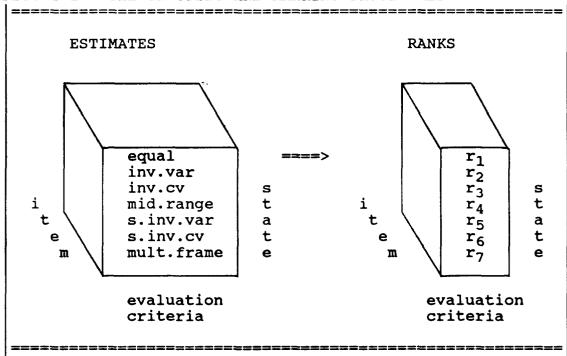
rmse =
$$\sqrt{\text{mse}}$$
 and std = $\sqrt{\text{var}}$.

Estimates were computed for each of the four evaluation criteria, each of the composites, and each hog category at the eight-state aggregate level for the two methods and at the state level for the eight states. The grouping criteria used in the eight-state aggregate analyses were: the two methods of computing the eight-state aggregate estimate, the four evaluation criteria, and the eight item categories. The grouping criteria used in the state analyses: the eight states, the four evaluation criterion, and the eight item categories.

Grouping the aggregate estimates resulted in the creation of 64 categories (four evaluation criteria times eight items times two methods). Similarly, the grouping of the state estimates produced 256 categories (four evaluation criteria times eight items times eight states). Each state hog category contained seven estimates, that is, one estimate for each of the seven composites. The generalized grouping depicted in the left half of Figure 1 displays those categories for the state level estimates. Replacing state with method in Figure 1 would depict the grouping of the aggregate level estimates.

The data was transformed to ranks within each grouping category for the analyses as depicted in Figure 1.

FIGURE 1 THE GROUPING AND RANKING PROCEDURES



Within each of the 256 groups the seven state level estimates were ranked from smallest to largest and then transformed to their respective ranks for analysis.

Transforming to ranks treats all states the same; treats all items the same; places equal importance on each of the four evaluation criteria; considers only order important within any group of seven estimates; and ignores all differences in magnitude between groups of seven estimates. In this sense the aggregate and state-by-state analyses were nonparametric.

The discussion of the aggregate and state analyses continues in the remainder of this section. Tables E.2 and E.3 of Appendix E summarize the state-by-state analyses for each of the eight states. Tables E.4 and E.5 of Appendix E summarize the item-by-item analyses for each of the eight hog categories. Table E.6 of Appendix E summarizes the analyses of average rank over all items and states.

Tables E.1, E.2, E.4, and E.6 use two asterisks next to the smallest mean rank (**) in each category (state or item) to aid in locating the best composite for that category.

Tables E.1, E.3, E.5, and E.6 display a set of digits under the column heading DIFFERENT that denote the composites that are significantly different from the composite listed under the row heading.

All the analyses used the same method of averaging the ranks over one or more classification variables and then testing for significant differences among the averages. Conover and Iman (2) describe this method of transforming the original data to ranks and analyzing the ranks' data by standard multivariate and univariate analysis techniques to produce nonparametric tests. The analyses presented in this report are for the most part similar to Friedman nonparametic analyses.

Modeling Interpretation

This section concentrates exclusively on the total hog series. Six ASB series were analyzed: first state recommendation, first ASB estimate, first state recommended revision, first ASB revision, second state recommended revision, and second ASB revision. Each series was modeled as a composite estimator with constant weights (w_T , w_F , w_W , w_{MF}) and a random error term. The modeling procedure used minimum mean square error as the criterion to choose the weights.

The six composite series (equal, inv.var, and so forth) were modeled in an analogous manner. Again, each series was modeled as the sum of a composite estimator with constant weights and a random error term.

An interesting question follows from the above discussion: How close are the models for the composite series to those for the ASB series? One method of quantifying the difference between two models is by calculating the four-dimensional Euclidian distance between their respective weights. The next section provides the details of such calculations.

Using four-dimensional Euclidian distance as the metric, Table 6 presents the number of states for which each of the composite models was closest to the indicated ASB model. Table 7 reveals which composite model for the eight-state aggregate was closest to the indicated ASB model.

TABLE 6 THE NUMBER OF STATES FOR WHICH A COMPOSITE MODEL WAS CLOSEST TO THE INDICATED ASB MODEL.

			asb M	ODEL			
COMPOSITE MODEL	1	2	3	4	<u>5</u>	<u> </u>	SUM
equal							
inv.var	5	2	2	2	2	2	15
inv.cv							
mid.range	1	1	1	1			4
s.inv.var	2	5	5	5	5	5	27
s.inv.cv					1	1	2

Note 1: Iowa corresponds to the 1's in the body of the table.

Note 2: The First State Recommendation, First ASB Estimate, First State Recommended Revision, First ASB Revision, Second State Recommended Revision, and Second ASB Revision are denoted by ASB 1, 2, 3, 4, 5, and 6 respectively.

TABLE 7 THE COMPOSITE MODEL FOR THE EIGHT-STATE
AGGREGATE CLOSEST TO THE INDICATED ASB MODEL.

AGGRE	BATE C	losest 	TO	THE	INDIC	ATED =====	ASB X	ODEL.
				<u>ASB</u>	MODEL			
COMPOSITE MODE	EL	1	2	3	4	<u>5</u>	<u>6</u>	SUM
equal inv.var inv.cv							*	1
<pre>mid.range s.inv.var</pre>			*	*	*			3
s.inv.cv		*				*		2

Note 1: The asterisks (*) denotes the composite model that is closest to the specified ASB model.

Note 2: Same as Note 2, Table 6

Table 6 shows that the ASB has treated all states except Iowa in a similar manner. The best model for each state except Iowa was either the inverse variance composite model or the smoothed inverse variance composite model. The choice between these two models is not important since these two composites are very similar. Additional tables in Appendix F further confirm that the ASB's treatment of states has been closest to the inverse variance composite for some states and the smoothed inverse variance composite for others except for Iowa.

Table 7 shows that the eight-state aggregate ASB average for eight years is closest to the mid.range composite model. The implication is that the mid.range model more accurately follows the eight-state aggregate ASB estimates than do the other composite models.

The apparent contradiction between Tables 5 and 6 required a more detailed examination of the eight individual states to establish the reason for the mid.range being best on the eight-state aggregate level while the inv.var and s.inv.var were closest for most of the individual states (see Tables F.1 through F.9, Appendix F for each state). The similarity between the eight-state aggregate results and those in Iowa and the dissimilarity between the eight-state aggregate results and those of the seven other individual states suggested that Iowa dominates the eight-state aggregate results.

Further examination of how Iowa dominated the eight-state aggregate required the computation of the weighted distance from the second revised eight-state aggregate composite model to each of the second revised state composite models. These weighted distances (see Table F.11, Appendix F) showed that the s.inv.var was the best model for the eight-state model when considering weighted distances. As a result, Iowa, which has more than one third of the eight-state aggregate, dominated the eight-state model when considering standard Euclidian distances. Iowa's importance in this analysis came about because the individual state distances between models were proportional to the state's total hog population. In summary, these model based interpretations of historical ASB estimates were consistent with the two sets of analyses presented earlier.

Details of the Model Interpretation

This section elaborates on the model interpretation methods in the previous ASB estimates section. The intent is to provide a more thorough description of both the modeling procedure and interpretations of the analyses.

Let the actual population value at time t be y_t and the value of the indication I be y_{It} . The elements of the estimated mean square error matrix (mse_{IJ}) are the following:

$$mse_{IJ} = 1/8 * \Sigma_{t=1}^{8} (y_{It} - y_t) (y_{Jt} - y_t)$$

where t = 1, ..., 8; I = T, F, W, MF (that is, the Tract, Farm, Weighted, and Multiple Frame indications, respectively); and J = T, F, W, MF.

The fundamental theorem on composite estimation (see Appendix A, Theorem 1) gives the constant weights applied to the tract, farm, weighted and the multiple frame indications that minimize the mean square error between the composite and the population values:

$$\mathbf{w} = \mathbf{MSE}^{-1} \mathbf{*e} / \mathbf{e}^{!} \mathbf{*MSE}^{-1} \mathbf{*e}$$

where
$$\underline{\mathbf{e}}' = (1, 1, \dots, 1)'$$
.

Substituting an ASB series for the actual population values in the discussion above produces a minimal mean square error model for the given ASB series. Similarly, substituting a composite series for the actual population values produces a minimal mean square error model for the given composite series. Each of these models is like a black box (a state space model with error) that can take the four indications as input and output an ASB or a composite estimate. The diagram in Figure 2 summarizes these ideas.

FIGURE 2 THE BEST MODEL FOR AN ASB ESTIMATE OR COMPOSITE ESTIMATE

INPUT	(W	r, w _F , w _W , w _{MF})	ij	OUTPUT
Tract Farm Weighted Multi.frame	===>	Model For State i And Output j	===>	ASB j or Composite j

These models determine one set of constant weights $(w_T, w_F, w_W, w_{MF})_{ij}$ for all eight years.

The formulas that calculate the distance between the two models and the formula to determine the composite model nearest to a given ASB model follow. Let the estimated fixed weights for ASB estimate b and composite c in state i be the following:

$$\underline{\mathbf{w}}_{ib} = (\mathbf{w}_{T}, \mathbf{w}_{F}, \mathbf{w}_{W}, \mathbf{w}_{MF})_{ib}$$
 and $\underline{\mathbf{w}}_{ic} = (\mathbf{w}_{T}, \mathbf{w}_{F}, \mathbf{w}_{W}, \mathbf{w}_{MF})_{ic}$

where i= 1,...,8; b= 1,...,6; and c= 1,...,8.
Then, the distance between the two models is the following:

$$d = dist(\underline{w}_{ib},\underline{w}_{ic}) = ||\underline{w}_{ib} - \underline{w}_{ic}||$$

where i= 1,...,8; b= 1,...,6; c= 1,...,8; and where the length of the four dimensional vector $\underline{\mathbf{w}}_{ib} - \underline{\mathbf{w}}_{ic}$ is represented by $||\underline{\mathbf{w}}_{ib} - \underline{\mathbf{w}}_{ic}||$.

The composite nearest or closest to a given ASB estimate B in state i is that composite having model parameters which are at a minimum distance to the ASB model parameters. The formula that determines the closest composite to the ASB estimate B is the following:

$$d_0 = \min_{\{c\}} \operatorname{dist}(\underline{\mathbf{w}}_{iB}, \underline{\mathbf{w}}_{ic}) = \min_{\{c\}} \operatorname{minimum} ||\underline{\mathbf{w}}_{iB} - \underline{\mathbf{w}}_{ic}||$$

where $i=1,\ldots,8$; $b=1,\ldots,6$; and $c=1,\ldots,8$; and d_0 is the minimal distance.

CONCLUSIONS

Three methods of analysis have examined the six composites during an eight year period for the eight largest hog-producing states. Each method has displayed new insights into how the composites compare with the ASB estimates.

The multivariate analyses have shown that the states vary on which composite most consistently is nearest to the ASB estimates. In examining the aggregate totals, the multivariate analyses showed that Iowa, which stays close to the midrange, made a significant contribution to making the aggregate total closest to the midrange as well. Finally, the multiple frame indicator and all the composites containing the multiple frame indicator (multi.frame, inv.var, and s.inv.var) did well in being least biased in relation to the ASB estimates.

Interpretation of the nonparametric analyses permitted the choice of the s.inv.var as being the "best" composite for the four criteria. Although the criteria did not individually make this conclusion, use of the combined four criteria made this selection possible.

Interpretation of the modeling analyses did not permit choosing a best compositing procedure. However, it shows that when all models are assumed to have constant weights then s.inv.var and inv.var composites have models that are closest to the ASB models. Since those two composites are very similar in theory, choosing the smooth inverse variance composite would still be reasonable.

Interpretation of the above analyses made possible the choice of the smooth inverse variance as the "best" composite under varied criteria and methods of analysis. Since the smoothed inverse variance composite is closest to the optimally weighted composite, this selection is reasonable.

RECOMMENDATIONS

The smoothed inverse variance composite (s.inv.cv) should be adopted as the ten state aggregate estimate for total hogs and pigs in the June survey, since the analyses of this study have shown that the smoothed inverse variance composite most closely reproduced the ASB results. Theoretically, the smoothed inverse variance composite should be closest to the optimal composite as well.

Sufficient data collection to compute the smoothed inverse variance composite for all states in June would permit the development of a national s.inv.var composite and statistical balance sheet estimate. These data would also aid research efforts on a statistically based revision and allocation process that would be less reliant on expert judgment.

The other three quarters have insufficient data to construct the four indications for the smoothed inverse variance composite. Therefore, additional research is necessary to relate the smoothed inverse variance composite to the hog series estimates for those quarters. Having all four indications and their estimated covariances in June for all states would help in developing a true national balance sheet with statistical properties. Furthermore, having estimates of the covariances between the individual indications would permit estimating the error term of the composite in June.

As part of the research analyses of the additional three quarters of the hog series, cost evaluations of the additional computer programming and processing required for the above recommendations are necessary. These cost figures would permit a determination of whether the proposed changes would be cost effective.

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APPENDIX A

COMPOSITE ESTIMATORS AND THEIR TRUE VARIANCES

INTRODUCTION

This appendix discusses the effects of substituting the estimated variance-covariance structure for the actual variance-covariance structure in the formula for the weights of the classical optimal composite estimator. The classical theorem gives the composite estimator as a function of the actual variance-covariance structure of the estimators when sampling from a static population. In NASS applications, the populations of interest are dynamic and the variance-covariance structure of the component estimators come from current or past survey data.

Formulas are derived for the variance of the classical composite estimator when only estimates of the variance-covariance structure of the component estimators are available under the assumption of multivariate normality. These formulas suggest that inferences based on classical composite estimation theory may be quite misleading when the component estimators are biased and instable covariance matrix estimates are used to derive the weights. The classical composite can be far from optimal when the estimators are biased. Hence, simple linear combinations may provide more reliable composite estimates than those based on the classical theory for some applications.

NASS calculates several estimates for some items of interest; for example, tract, farm, weighted, and multiple frame estimates for total hogs from the June survey data. Sampling and nonsampling errors prevent these estimates from having the same numerical value. Since each estimate contains information about the item of interest, combining the estimates into a composite estimate should be desirable. The usual way of obtaining a composite is to form a linear combination of the individual estimates with either minimal variance or minimal mean square error.

The first section of this appendix shows that the optimal linear composite is a function of the variance-covariance structure of the sampled population and the estimators. The second section shows how the dynamic nature of the population may cause the classical composite estimation theory to be quite misleading in determining the population's structure from current or past survey data. The third section focuses on the NASS hog series to provide

additional insights into the nature of the difficulties of applying composite estimation.

THE CLASSICAL COMPOSITE ESTIMATION THEOREM

The idea underlying composite estimation is the following: Given n estimators Y_1, Y_2, \ldots, Y_n of a population parameter θ , find a set of weights w_1, w_2, \ldots, w_n such that the linear combination $Y = w_1Y_1 + w_2Y_2 + \ldots + w_nY_n$ is best or optimal in some significant statistical sense.

Theorem 1: Suppose that $\underline{Y} = (Y_1, Y_2, \ldots, Y_n)^T$ are n estimators of a population parameter θ with covariance matrix $\Sigma = (\sigma_{ij})$. Further suppose that $\underline{b} = (b_1, b_2, \ldots, b_n)^T$ is the vector of biases associated with these estimators. Then, for $\underline{w}^T\underline{e} = 1$, the best (minimal mean square error) linear composite estimator of θ is the following:

$$Y_c = \underline{w}^T \underline{x} = w_1 Y_1 + w_2 Y_2 + \dots + w_n Y_n$$
 (Equation 1.1)

where

$$\mathbf{W} = (\mathbf{w}_1, \mathbf{w}_2, \dots, \mathbf{w}_n)^T = \mathbf{A}^{-1} \underline{\mathbf{e}} / \underline{\mathbf{e}}^T \mathbf{A}^{-1} \underline{\mathbf{e}},$$

$$\mathbf{A} = \mathbf{\Sigma} + \underline{\mathbf{b}} \underline{\mathbf{b}}^T, \text{ and } \underline{\mathbf{e}} = (1, 1, \dots, 1)^T.$$

The mean square error of the composite estimate $\mathbf{Y}_{\mathbf{C}}$ is as follows:

$$Mse(Y_C) = \underline{\mathbf{w}}^T \mathbf{A} \underline{\mathbf{w}} = \underline{\mathbf{w}}^T (\mathbf{\Sigma} + \underline{\mathbf{b}}\underline{\mathbf{b}}^T) \underline{\mathbf{w}} = 1/\underline{\mathbf{e}}^T \mathbf{A}^{-1}\underline{\mathbf{e}}.$$
 (Equation 1.2)

The proof of this result is well known (see references (3) and (6)).

THE EFFECT OF ESTIMATING THE WEIGHT VECTOR W ON THE COMPOSITE ESTIMATORS

The population parameter Σ and the biases \underline{b} are unknown in practice and require estimation from current or past survey data. This means that in the composite estimate $Y_C = \underline{w}^T\underline{y}$ both \underline{w} and \underline{y} are random variables, since they both must be estimated by the data. There are two implications to this observation. First, since the classical theorem does not consider \underline{w} to be a random variable, the composite given in Equation 1.1 does not necessarily minimize the mean square error, $\operatorname{Mse}(Y_C)$. Second, since the composite contains a component of variation because of the randomness of \underline{w} and \underline{y} , the formula in Equation 1.2 for the mean square error will underestimate the true variance of the composite.

Theorem 2. Suppose that the vector of sample statistics ($\underline{\mathbf{Y}}$) comes from a multivariate normal distribution and replacing the elements of $\Sigma_{\underline{\mathbf{Y}}}$ with their sample estimates will provide the optimal weights in Theorem 1. Then, the variance of the composite, $\mathbf{Y}_{\underline{\mathbf{C}}} = \underline{\mathbf{W}}^T\underline{\mathbf{Y}}$, is the following:

$$V(Y_{C}) = \underline{\mu}_{W}^{T} \Sigma_{Y} \underline{\mu}_{W} + \underline{\mu}_{Y}^{T} \Sigma_{W} \underline{\mu}_{Y} + tr(\Sigma_{W} \Sigma_{Y})$$
 (Equation 2.1)

where

$$\underline{\mu}_{\mathbf{W}} = \mathbf{E}(\underline{\mathbf{Y}})$$
 and $\underline{\mu}_{\mathbf{Y}} = \mathbf{E}(\underline{\mathbf{Y}})$.

Moreover, all three terms on the right hand side of Equation 2.1 are non-negative.

Proof: Standard theorems on the variance operator permit writing the unconditional dispersion of Y_C as equal to the dispersion with respect to \underline{Y} of the conditional expectation of Y_C given \underline{Y} plus the expected value with respect to \underline{Y} of the conditional dispersion of Y_C given \underline{Y} as follows:

$$\begin{split} V(Y_{\mathbf{C}}) &= V_{\mathbf{Y}}[E_{\mathbf{W} \mid \mathbf{Y}}(Y_{\mathbf{C}})] + E_{\mathbf{Y}}[V_{\mathbf{W} \mid \mathbf{Y}}(Y_{\mathbf{C}})] \\ &= V_{\mathbf{Y}}[E_{\mathbf{W} \mid \mathbf{Y}}(\underline{\mathbf{w}}^{T}\underline{\mathbf{Y}})] + E_{\mathbf{Y}}[V_{\mathbf{W} \mid \mathbf{Y}}(\underline{\mathbf{w}}^{T}\underline{\mathbf{Y}})]. \end{split}$$

Since the assumption is that the sample comes from a multivariate normal population, the elements of the sample

dispersion matrix (and hence the weights \underline{w}) are independent of the sample mean of \underline{v} . Thus

$$E_{\mathbf{W} \mid \mathbf{Y}}(\mathbf{\underline{w}}^{T}\mathbf{\underline{Y}}) = \underline{\mu}_{\mathbf{W}}^{T}\mathbf{\underline{Y}}$$
 and
$$V_{\mathbf{W} \mid \mathbf{Y}}(\mathbf{\underline{w}}^{T}\mathbf{\underline{Y}}) = \mathbf{\underline{Y}}^{T}\boldsymbol{\Sigma}_{\mathbf{W}}\mathbf{\underline{Y}}$$
 (Equations 2.2)

then

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$$V(Y_C) = V_Y(\underline{\mu}_W^T \underline{Y}) + E_Y(\underline{Y}^T \underline{\Sigma}_W \underline{Y})$$

$$= \underline{\mu}_W^T \underline{\Sigma}_Y \underline{\mu}_W + E_Y(\underline{Y}^T \underline{\Sigma}_W \underline{Y}).$$

Evaluation of the second term in the above expression involves standard but rather intricate matrix theory which is presented below. The result is that the variance of the composite consists of three components:

- 1. $\underline{\mu}_{W}^{T} \Sigma_{Y} \underline{\mu}_{W}$, the variance of the composite when \underline{w} is held fixed at its mean $\underline{\mu}_{W}$,
- 2. $\underline{\mu}_{Y}^{T} \Sigma_{W} \underline{\mu}_{Y}$, the variance of the composite when \underline{Y} is held fixed at its mean $\underline{\mu}_{Y}$, and,
- 3. $Tr(\Sigma_w\Sigma_v)$, the sum of the diagonal elements of the product of the dispersion matrices for \underline{x} and \underline{w} , which is greater than or equal to zero.

Using the properties of the trace and expected value operators:

$$\begin{split} \mathbf{E}_{\mathbf{Y}}(\underline{\mathbf{Y}}^{\mathbf{T}}\mathbf{\Sigma}_{\mathbf{W}}\underline{\mathbf{Y}}) &= \mathbf{E}_{\mathbf{Y}}[\mathbf{tr}(\underline{\mathbf{Y}}^{\mathbf{T}}\mathbf{\Sigma}_{\mathbf{W}}\underline{\mathbf{Y}})] \\ &= \mathbf{E}_{\mathbf{Y}}[\mathbf{tr}(\mathbf{\Sigma}_{\mathbf{W}}\underline{\mathbf{Y}}\underline{\mathbf{Y}}^{\mathbf{T}})] \\ &= \mathbf{tr}[\mathbf{E}_{\mathbf{Y}}(\mathbf{\Sigma}_{\mathbf{W}}\underline{\mathbf{Y}}\underline{\mathbf{Y}}^{\mathbf{T}})] \\ &= \mathbf{tr}(\mathbf{\Sigma}_{\mathbf{W}}[\mathbf{E}_{\mathbf{Y}}(\underline{\mathbf{Y}}\underline{\mathbf{Y}}^{\mathbf{T}})] \\ &= \mathbf{tr}[\mathbf{\Sigma}_{\mathbf{W}}(\mathbf{\Sigma}_{\mathbf{Y}} + \underline{\mu}_{\mathbf{Y}}\underline{\mu}\underline{\mathbf{Y}}^{\mathbf{T}})] \\ &= \mathbf{tr}(\mathbf{\Sigma}_{\mathbf{W}}\mathbf{\Sigma}_{\mathbf{Y}}) + \mathbf{tr}(\mathbf{\Sigma}_{\mathbf{W}}\underline{\mu}_{\mathbf{Y}}\underline{\mu}\underline{\mathbf{Y}}^{\mathbf{T}}) \\ &= \mathbf{tr}(\mathbf{\Sigma}_{\mathbf{W}}\mathbf{\Sigma}_{\mathbf{Y}}) + \mathbf{tr}(\underline{\mu}_{\mathbf{Y}}^{\mathbf{T}}\mathbf{\Sigma}_{\mathbf{W}}\underline{\mu}_{\mathbf{Y}}) \\ &= \mathbf{tr}(\mathbf{\Sigma}_{\mathbf{W}}\mathbf{\Sigma}_{\mathbf{Y}}) + \underline{\mu}_{\mathbf{Y}}^{\mathbf{T}}\mathbf{\Sigma}_{\mathbf{W}}\underline{\mu}_{\mathbf{Y}}. \end{split}$$

Showing that the trace of the product $\Sigma_w \Sigma_Y$ is greater than or equal to zero completes the proof. Although this proof follows from Graybill (4), page 307, Theorem 9.1.28, a proof follows below.

Since Σ_W and Σ_Y are dispersion matrices, both matrices are symmetric and non-negative. Thus, there exist orthogonal matrices P and Q and diagonal matrices $\Gamma = (\tau_1)$ and $D = (\delta_1)$ such that

$$\boldsymbol{\Sigma}_{\mathbf{W}} = \mathbf{P} \mathbf{\Gamma} \mathbf{P}^{\mathbf{T}} \quad \text{and} \quad \boldsymbol{\Sigma}_{\mathbf{Y}} = \mathbf{Q} \mathbf{D} \mathbf{Q}^{\mathbf{T}}$$
where $\mathbf{P} \mathbf{P}^{\mathbf{T}} = \mathbf{P}^{\mathbf{T}} \mathbf{P} = \mathbf{I}$, $\mathbf{Q} \mathbf{Q}^{\mathbf{T}} = \mathbf{Q}^{\mathbf{T}} \mathbf{Q} = \mathbf{I}$, $\boldsymbol{\tau}_{N} \geq \boldsymbol{\tau}_{N-1} \geq \ldots \geq \boldsymbol{\tau}_{1}$, and $\boldsymbol{\delta}_{N} \geq \boldsymbol{\delta}_{N-1} \geq \ldots \geq \boldsymbol{\delta}_{1}$.

Of course the columns \underline{P}_i of P are the eigenvectors corresponding to the eigenvalues τ_i of Σ_W and the columns \underline{Q}_i of \underline{Q} are the eigenvectors corresponding to the δ_i of Σ_Y . Using the above decomposition of Σ_W and Σ_Y and properties of the trace operator:

$$tr(\Sigma_{\mathbf{w}}\Sigma_{\mathbf{Y}}) = tr(\mathbf{P}\mathbf{\Gamma}\mathbf{P}^{\mathbf{T}}\Sigma_{\mathbf{Y}})$$

$$= tr(\Sigma_{\mathbf{i}}\tau_{\mathbf{i}}\mathbf{P}_{\mathbf{i}}\mathbf{P}_{\mathbf{i}}^{\mathbf{T}}\Sigma_{\mathbf{Y}})$$

$$= \Sigma_{\mathbf{i}}\tau_{\mathbf{i}}tr(\mathbf{P}_{\mathbf{i}}\mathbf{P}_{\mathbf{i}}^{\mathbf{T}}\Sigma_{\mathbf{Y}})$$

$$= \Sigma_{\mathbf{i}}\tau_{\mathbf{i}}tr(\mathbf{P}_{\mathbf{i}}^{\mathbf{T}}\Sigma_{\mathbf{Y}}\mathbf{P}_{\mathbf{i}})$$

$$= \Sigma_{\mathbf{i}}\tau_{\mathbf{i}}(\mathbf{P}_{\mathbf{i}}^{\mathbf{T}}\mathbf{Q}\mathbf{D}\mathbf{Q}^{\mathbf{T}}\mathbf{P}_{\mathbf{i}})$$

$$= \Sigma_{\mathbf{i}}\tau_{\mathbf{i}}(\mathbf{P}_{\mathbf{i}}^{\mathbf{T}}(\Sigma_{\mathbf{j}}\delta_{\mathbf{j}}\mathbf{Q}_{\mathbf{j}}\mathbf{Q}_{\mathbf{j}}^{\mathbf{T}})\mathbf{P}_{\mathbf{i}})$$

$$= \Sigma_{\mathbf{i}}\tau_{\mathbf{i}}\Sigma_{\mathbf{j}}\delta_{\mathbf{j}}\mathbf{P}_{\mathbf{i}}^{\mathbf{T}}\mathbf{Q}_{\mathbf{j}}\mathbf{Q}_{\mathbf{j}}^{\mathbf{T}}\mathbf{P}_{\mathbf{i}}.$$

Since $Q_jQ_j^T$ is a projection, the eigenvalues of $Q_jQ_j^T$ are either zero or one. Hence,

$$0 = \inf_{\substack{|\underline{\mathbf{x}}| = 1}} \underline{\mathbf{x}}^{\mathrm{T}}(\underline{Q}_{j}\underline{Q}_{j}^{\mathrm{T}})\underline{\mathbf{x}} \leq \underline{\underline{\mathbf{P}}}_{1}^{\mathrm{T}}(\underline{Q}_{j}\underline{Q}_{j}^{\mathrm{T}})\underline{\underline{\mathbf{P}}}_{1} \leq \sup_{\substack{|\underline{\mathbf{x}}| = 1}} \underline{\mathbf{x}}^{\mathrm{T}}(\underline{Q}_{j}\underline{Q}_{j}^{\mathrm{T}})\underline{\mathbf{x}} = 1$$

for all i and j. Then, the next equation follows:

$$0 \le \operatorname{tr}(\Sigma_{\mathbf{w}}\Sigma_{\mathbf{Y}}) \le \Sigma_{\mathbf{1}=\mathbf{1}}^{\mathbf{N}}\Sigma_{\mathbf{j}=\mathbf{1}}^{\mathbf{N}}\tau_{\mathbf{i}}\delta_{\mathbf{j}} \le N^{2}.$$

Q.E.D.

The multivariate normal assumption was used only to justifies Equation 2.2 in the proof of Theorem 2. Since requiring the independence of the weights \underline{w} and the sample mean of $\underline{\underline{v}}$ is in general more restrictive than requiring Equation 2.2, a somewhat stronger theorem is possible.

Theorem 3. Suppose that the following equations hold

Then, the variance of the composite, $Y_C = \underline{\mathbf{w}}^T \underline{\mathbf{x}}$, is the following:

$$V(Y_{C}) = \underline{\mu}_{W}^{T} \Sigma_{Y} \underline{\mu}_{W} + \underline{\mu}_{Y}^{T} \Sigma_{W} \underline{\mu}_{Y} + \text{tr}(\Sigma_{W} \Sigma_{Y})$$
 (Equation 3.2)

where

$$\underline{\mu}_{W} = E(\underline{Y})$$
 and $\underline{\mu}_{Y} = E(\underline{Y})$.

Moreover, all three terms on the right hand side of equation 3.2 are non-negative.

No proof is given because the proof is very similar to the one for Theorem 2.

AN EXAMPLE: THE HOG SERIES

This section examines the variance of the classical composite estimate for the NASS hog series. This example will provide some insights into the difficulties encountered with composite estimation when estimating the weights from current or past survey data.

Denote the tract, farm, weighted, and multiple frame indications, their associated covariance matrix, and the covariance matrix of the composite weights as follows:

- 1. The indications by, $\mathbf{X} = (Y_1, Y_2, Y_3, Y_4)^T$,
- 2. The covariance matrix of \underline{Y} by $\Sigma_{\underline{Y}} = (\sigma_{ij})$, and,
- 3. The covariance of $\underline{\mathbf{w}}$ by $\Sigma_{\mathbf{w}} = (\pi_{ij})$.

Then the central limit theorem permits an approximation of the variance of the classical composite by Equation 2.1. Writing this out in full results in the following:

Since $w_1 + w_2 + w_3 + w_4 = 1$, the following set of relationships exist among the elements of the covariance matrix of w:

$$\pi_{14} = -\pi_{11} - \pi_{12} - \pi_{13}$$

$$\pi_{24} = -\pi_{21} - \pi_{22} - \pi_{23}$$

$$\pi_{34} = -\pi_{31} - \pi_{32} - \pi_{33}$$

$$\pi_{44} = -\pi_{41} - \pi_{42} - \pi_{43}$$

$$= -\pi_{11} + \pi_{22} + \pi_{33} + 2\pi_{12} + 2\pi_{13} + 2\pi_{23}.$$

Using these relationships allows rewriting the second term in the expression for the composite, which is the variance of the composite with $\underline{\mathbf{Y}}$ fixed at its mean $\underline{\mu}_{\mathbf{Y}}$, as follows:

Since this term represents only part of the extra variance of the classical composite which is due to estimating the weights (the other component is the $\operatorname{tr}(\Sigma_w\Sigma_Y)$) and since nonsampling renders the component indications biased the true variance of the classical composite can be much larger than the classical theorem suggest. Thus, simple linear combinations of the tract, farm, weighted, and multiple frame estimates may provide more reliable estimates than the classical composite.

APPENDIX B

GRAPHS OF INDICATIONS, COMPOSITES AND ASB ESTIMATES

The following graphical representations for each of the hog categories highlight the differences and similarities observed among the four indications (tract, farm, weighted, and multiple frame) and among the six composites for the eight-state aggregate. They span the years 1979 through 1986 and present only the June data.

The specific hog categories for the eight states in the study are those of total hogs, total breeding hogs, total market hogs, market hogs less than 60 pounds, market hogs 60-119 pounds, market hogs 120-179 pounds, and market hogs 180 pounds and larger. The first graph of each pair of graphs presents the four indications obtained from the June Enumerative Survey (JES), while the second graph shows the resulting composites. The final ASB value for each of the corresponding years provides a reference value.

Graph B.1.1 Total Hogs Indications

A - - - Today or - "A"

This graph presents the June indications for total hogs for the eight states under study during the years 1979-1986. The four indications vary in their relationship to the Board estimate. Note in particular that the weighted indication switches with the tract as the largest indication and that the multiple frame switches with the farm as the smallest indication.

Graph B.1.2 Total Hogs Composites

This graph illustrates the small differences among the six composites obtained from the indications for the total hogs category in the eight states. The mid.range is the composite most often distinguishable, however, the range of all the estimates is small.

Graph B.2.1 Total Breeding Hogs Indications

The relationships of the four indications follow the same pattern as that of the total hogs indications, including the indications which switch the high and low values. Again, the indications quite closely follow the ASB estimates.

Graph B.2.2 Total Breeding Hogs Composites

As for the case of the total hogs composites, the total breeding hogs composites follow within a small range of one

another. No composite appears prominently separate from the others.

<u>Graphs B.3.1 - B.3.2 Total Market Hogs</u> <u>Indications and Composites</u>

The relationships of the indications and composites for the total market hogs follow the same distribution as that of the total hogs.

Graphs B.4.1 - B.4.2 Market Hogs Less Than 60 lbs. Indications and Composites

All the indications for the market hogs less than 60 pounds follow a similar trend as do the total hogs for the composites. However, the ASB estimate continues higher than the composite for every year. The years 1984 through 1986 are the most noticeable portion of this trend.

Graphs B.5.1 - B.5.2 Market Hogs 60 - 119 lbs. Indications and Composites

The number of hogs in this category does not follow the general trends of the total hog category. Although the indications follow the same ordering as in the total hogs category, they show more dispersion. The ASB estimate, except for 1986, remains below the tightly bunched composites.

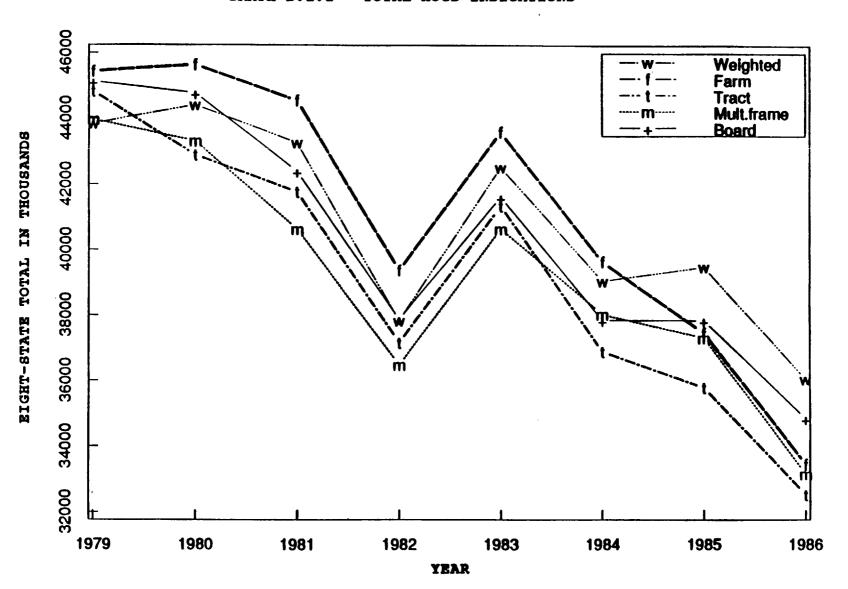
Graphs B.6.1 - B.6.2 Market Hogs 120 - 179 lbs. Indications and Composites

Although the ordering of the indications from smallest to largest remains similar to that of the total hogs, the trends exhibited by this category show much less variation than even that of the 60-119 category. The ASB estimate is always less than any of the composites.

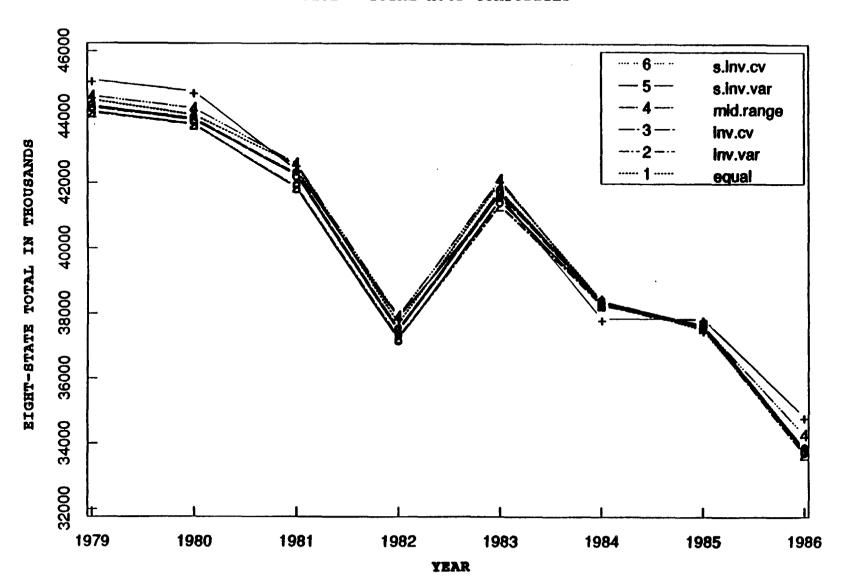
Graphs B.7.1 - B.7.2 Market Hogs 180 lbs. and Larger Indications and Composites

The market hogs 180 pounds and larger category indications and composites show nearly a constant value for the eight states during the eight years. Although the total number of hogs had dropped by nearly 12 million, the largest weight category has remained nearly the same. From 1984, the ASB estimates and the composites all agree very well.

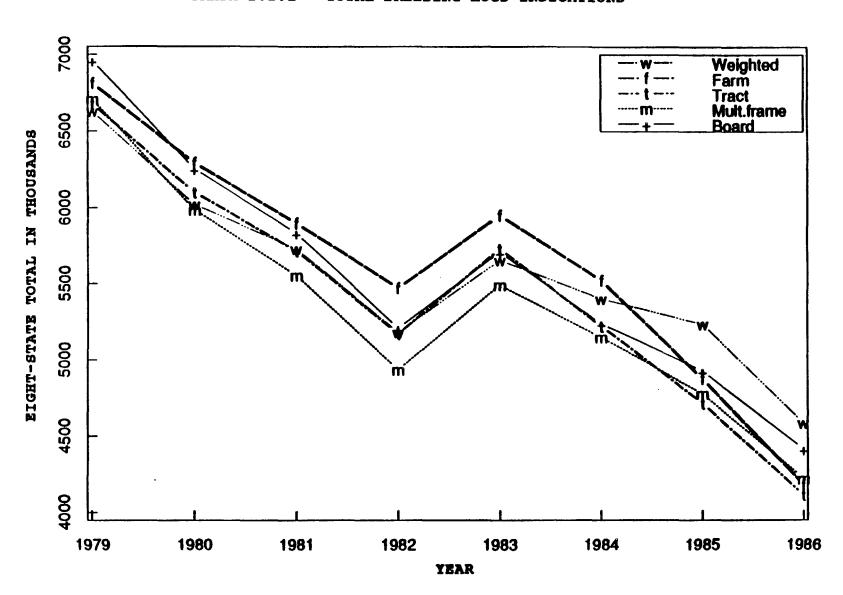
GRAPH B.1.1 TOTAL HOGS INDICATIONS



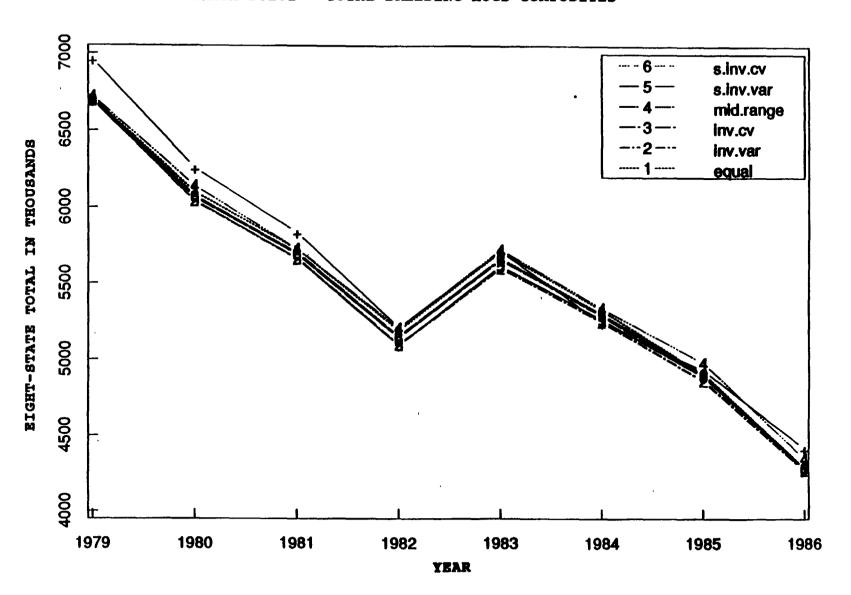
GRAPH B.1.2 TOTAL HOGS COMPOSITES



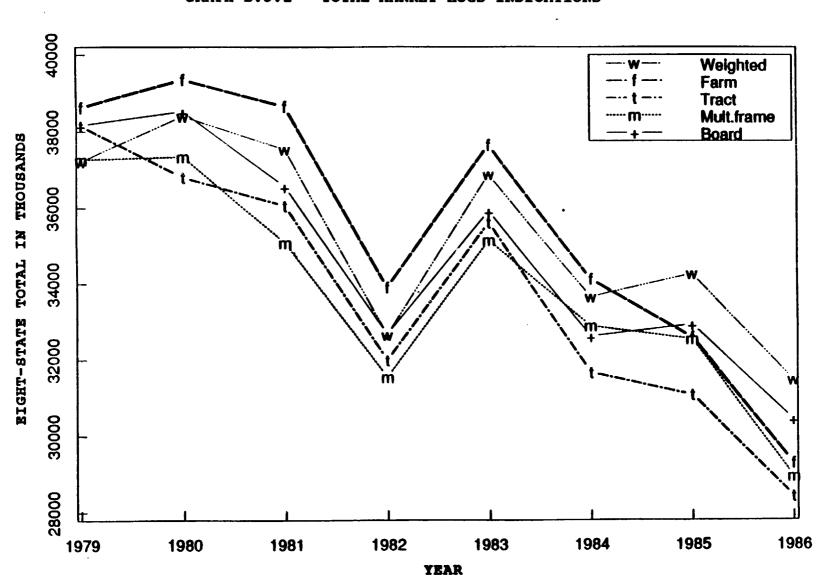
GRAPH B.2.1 TOTAL BREEDING HOGS INDICATIONS



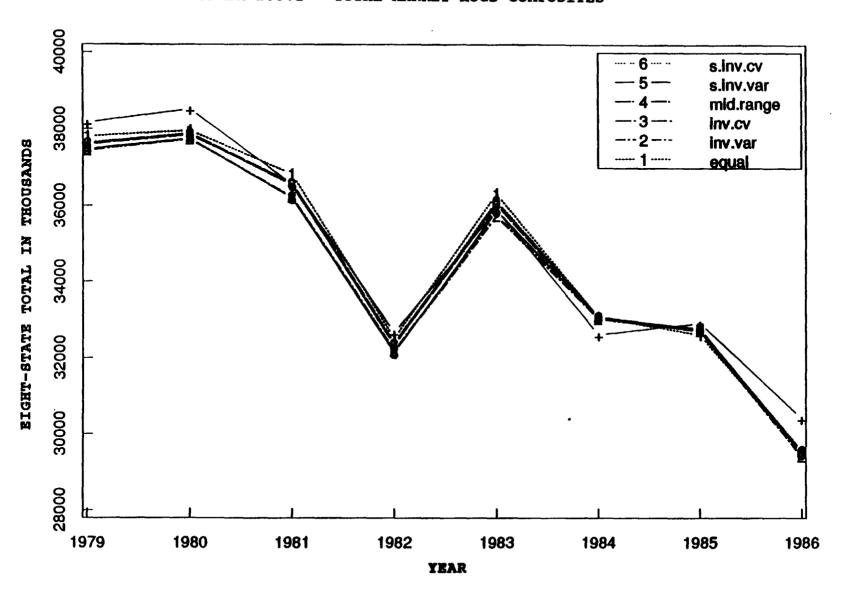
GRAPH B.2.2 TOTAL BREEDING HOGS COMPOSITES



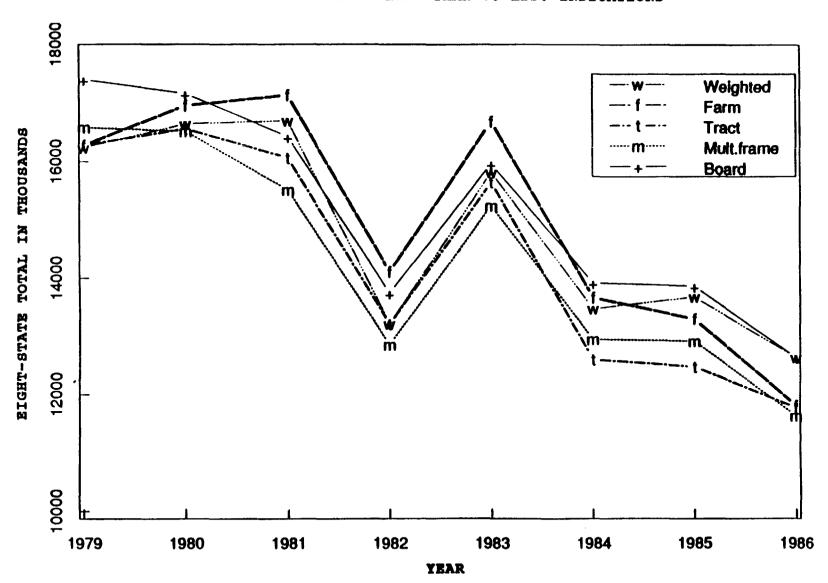
GRAPH B.3.1 TOTAL MARKET HOGS INDICATIONS



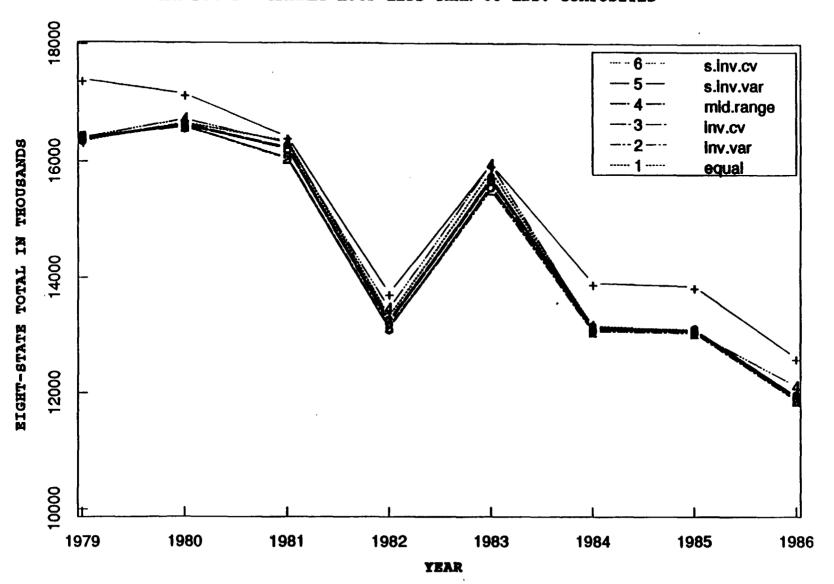
GRAPH B.3.2 TOTAL MARKET HOGS COMPOSITES



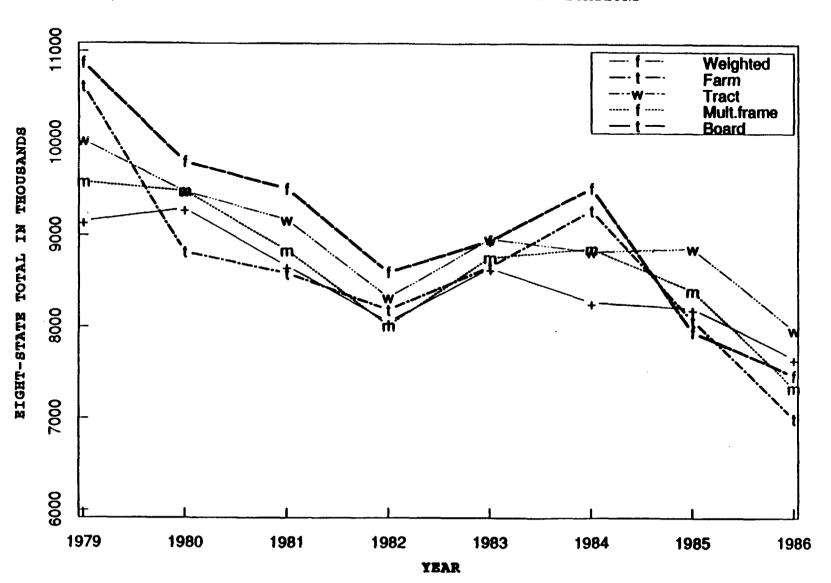
GRAPH B.4.1 MARKET HOGS LESS THAN 60 LBS. INDICATIONS



GRAPH B.4.2 MARKET HOGS LESS THAN 60 LBS. COMPOSITES

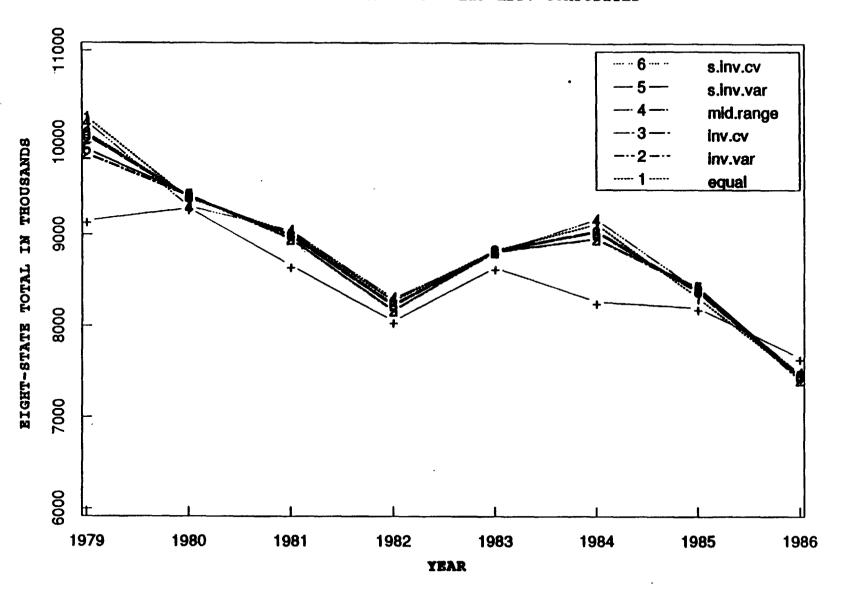


GRAPH B.5.1 MARKET HOGS 60 - 119 LBS. INDICATIONS

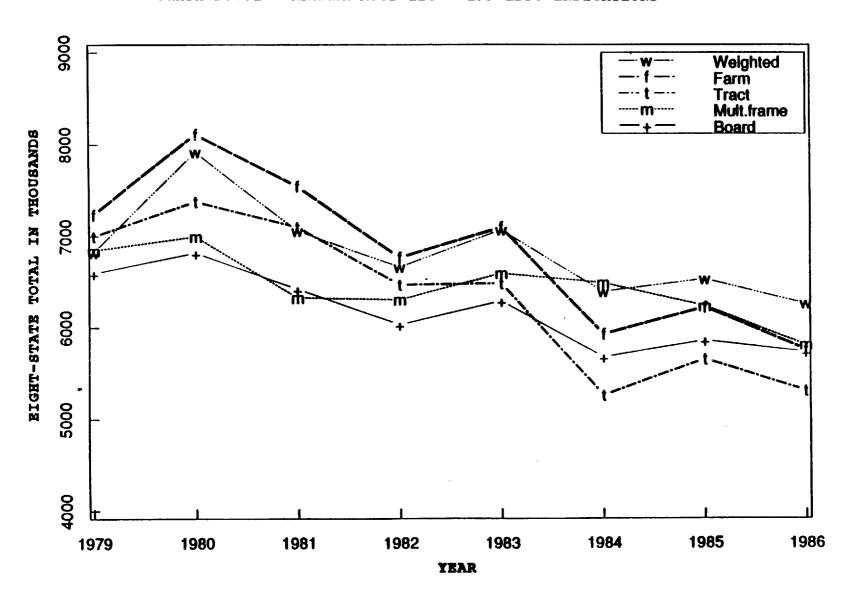


4

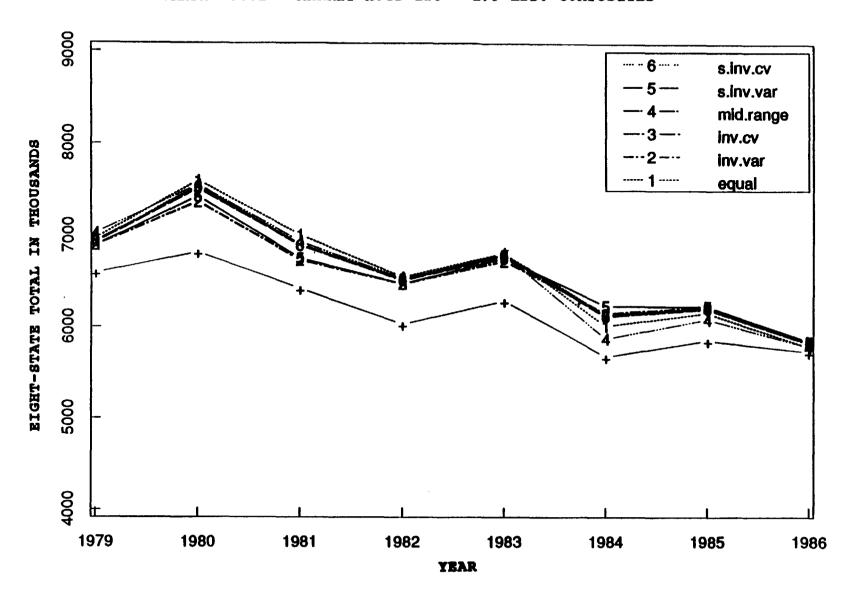
GRAPH B.5.2 MARKET HOGS 60 - 119 LBS. COMPOSITES



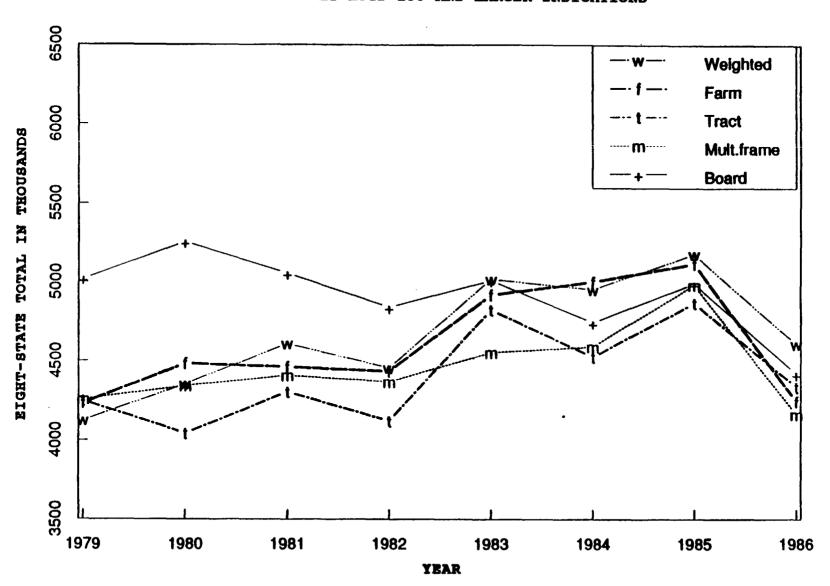
GRAPH B.6.1 MARKER HOGS 120 - 179 LBS. INDICATIONS



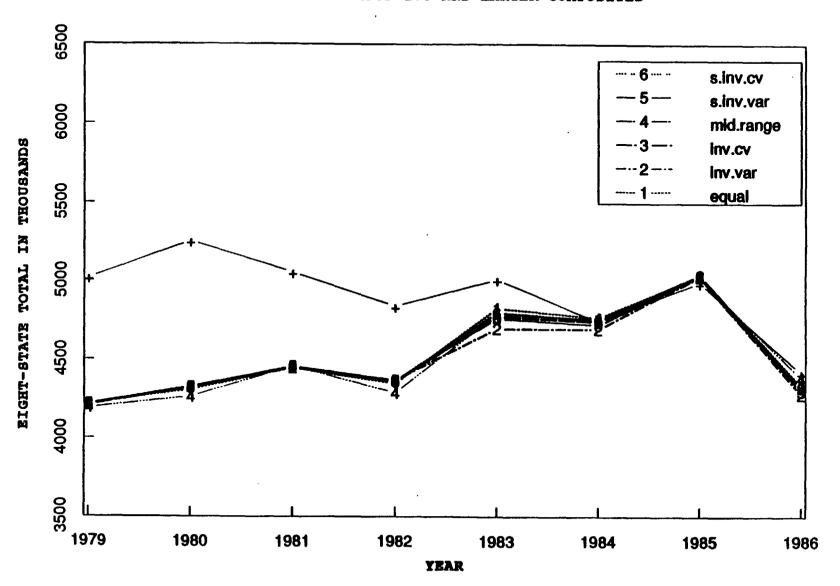
GRAPH B.6.2 MARKET HOGS 120 - 179 LBS. COMPOSITES



GRAPH B.7.1 MARKET HOGS 180 AND LARGER INDICATIONS



GRAPH B.7.2 MARKET HOGS 180 AND LARGER COMPOSITES



APPENDIX C

TABLES OF SUMMARY STATISTICS FOR THE WEIGHTS IN VARIABLE WEIGHT COMPOSITES

Four of the composites (the inverse variance, smoothed inverse variance, inverse coefficient of variation and smoothed inverse coefficient of variation composite) have weights that change over time. Summary statistics for the weights of each of these composites for the years 1979 - 1986 are displayed in this appendix.

Tables C.1, C.2, C.3 and C.4 provide summary statistics for the weights of the eight-state aggregate by item category for the four composites: the inv.var, s.inv.var, inv.cv and s.inv.cv composite. Tables C.5, C.6, C.7 and C.8 provide summary statistics for the weights of the total hogs by state for four composites.

A comparison of the inverse variance and inverse coefficient of variation weights tables with the corresponding smoothed inverse variance and inverse coefficient of variation weights tables (Tables C.1, C.3, C.5 and C.7 with Tables C.2, C.4, C.6 and C.8, respectively) indicated that the corresponding smoothed weights have much smaller coefficients of variation.

A comparison among item categories of mean inverse variance weights given in Tables C.1 indicated that (except for pig crop and deaths, for which only three indications are available) the weights are relatively stable among item categories. Similar conclusions can be obtained for the other three composites by comparing the weights given in Tables C.2, C.3 and C.4.

A comparison among states of total hogs mean inverse variance weights given in Table C.5 indicate large state to state variations. For example, the mean multiple frame and tract weights were (0.65 and 0.08) and (0.45 and 0.19) in Illinois and Ohio, respectively. Similar examples for the other three composites can be obtained by comparing the weights given in Tables C.6, C.7 and C.8.

TABLE C.1 SUMMARY STATISTICS FOR THE WEIGHTS OF THE INV.VAR COMPOSITE: TOTAL HOGS FOR EIGHT-STATES BY CATEGORY

<u>CATEGORY</u>	WEIGHT	MINIMUM	MAXIMUM	MEAN	COEFF. OF VARIATION
TOTAL HOGS	Wrp	0.099	0.138	0.122	12.3
	wr	0.079	0.108	0.096	10.1
	www.	0.200	0.294	0.242	15.4
	w_{MF}	0.472	0.617	0.540	8.5
BREED HOGS	$w_{f T}$	0.111	0.143	0.125	
	wr	0.094	0.125	0.106	
	ww	0.171	0.290	0.241	
	$w_{\mathbf{MF}}$	0.449	0.605	0.528	10.8
MARKET HOGS	Wդ	0.097		0.121	
	$\mathbf{w}_{\mathbf{F}}^{\perp}$	0.078		0.096	
	ww	0.204	0.295	0.245	
	$w_{MF}^{"}$	0.465	0.620	0.537	8.9
UNDER 60 LB	w _T	0.111	0.141	0.123	8.9
	wr	0.077	0.119	0.101	14.3
	ww	0.213	0.329	0.254	
	w_{MF}	0.423	0.590	0.523	10.8
60-119 LB	W _T	0.078	0.146	0.114	21.5
	wr	0.064	0.122	0.095	
	$w_{\overline{W}}$	0.189	0.299	0.248	
	w_{MF}	0.491	0.605	0.543	7.6
120-179 LB	₩Ţ	0.104	0.185		
	w _F	0.080			
	$\overline{w_W}$	0.217		0.264	
	w_{MF}	0.381	0.598	0.496	14.9
180 LB UP	$\mathbf{w}_{\mathbf{T}}$	0.079			
	$\mathbf{w_F^-}$	0.084	0.155	0.112	20.8
	ww	0.194	0.394	0.261	26.2
	$w_{\mathbf{MF}}$	0.264	0.635	0.504	23.9
BIRTHS	W _F	0.095	0.155	0.121	
	ww	0.219	0.367	0.291	
	$w_{\mathbf{MF}}$	0.509	0.671	0.588	9.8
DEATHS	W _F	0.027	0.278	0.156	55.3
	ww	0.028	0.448	0.325	
	$w_{MF}^{"}$	0.345	0.945	0.519	36.6

TABLE C.2 SUMMARY STATISTICS FOR THE WEIGHTS OF THE S.INV.VAR COMPOSITE: TOTAL HOGS FOR EIGHT-STATES BY CATEGORY

========	=======	=========			
					COEFF. OF
CATEGORY	WEIGHT	MINIMUM	MUMIXAM	MEAN	VARIATION
TOTAL HOGS	$\mathbf{w_{T}}$	0.121	0.129	0.123	2.2
	$\mathbf{w_F^1}$	0.088	0.097	0.093	3.2
	ww	0.088 0.230	0.282	0.261	7.4
	w_{MF}	0.499	0.549	0.523	3.6
BREED HOGS	$\mathbf{w}_{\mathbf{T}}$	0.123	0.137	0.129	3.8
	wr	0.098	0.106	0.101	2.4
	ww	0.224	0.299	0.270	10.5
	w_{MF}	0.465	0.546	0.500	6.3
MARKET HOGS	$\mathbf{w_{T}}$	0.118	0.128	0.122	2.6
	wr	0.089	0.098	0.094	
	ww	0.233	0.282	0.262	6.8
	w_{MF}	0.498	0.546	0.522	3.3
UNDER 60 LB	$w_{\mathbf{T}}$	0.123	0.132		
	$\mathbf{w}_{\mathbf{F}}^{-}$	0.093	0.104		
	ww	0.242	0.286	0.266	
	w_{MF}	0.479	0.536	0.509	4.1
60-119 LB	Wт	0.109	0.122	0.115	
	$\mathbf{w_F^-}$	0.091	0.104	0.096	
	ww	0.238	0.273	0.260	
	w_{MF}	0.516	0.551	0.529	2.1
120-179 LB	Wm	0.118	0.136	0.127	5.3
	$w_{ m F}^{-}$	0.097	0.116	0.104	
	$\overline{w_W}$	0.252	0.314	0.280	
	w_{MF}	0.458	0.517	0.489	3.5
180 LB UP	$w_{\mathbf{T}}$		0.148		
	wr	0.106	0.124	0.112	4.9
	w _w	0.243	0.308	0.274	7.7
	w_{MF}	0.420	0.532	0.486	6.8
BIRTHS	$w_{\mathbf{F}}$	0.117	0.130	0.123	3.8
	ww	0.281	0.360	0.324	8.3
	$\mathbf{w_{MF}}$	0.510	0.592	0.552	5.4
DEATHS	$w_{\mathbf{F}}$	0.120	0.190	0.168	14.9
	ww	0.279	0.371	0.342	9.2
	$\mathbf{w_{MF}}$	0.439	0.582	0.490	11.1

TABLE C.3 SUMMARY STATISTICS FOR THE WEIGHTS OF THE INV.CV COMPOSITE: TOTAL HOGS FOR EIGHT-STATES BY CATEGORY

	=======				
					COEFF. OF
CATEGORY	WEIGHT	MINIMUM	MAXIMUM	<u>MEAN</u>	<u>VARIATION</u>
TOTAL HOGS	$w_{\mathbf{T}}$	0.170	0.193	0.181	4.6
	wF	0.160	0.179	0.169	
	$w_{\overline{W}}$	0.252	0.286		
	WMF	0.349	0.418	0.383	5.9
BREED HOGS	$w_{\mathbf{T}}$	0.178	0.195		
	$\mathbf{w}_{\mathbf{F}}^{-}$	0.169	0.182	0.177	
	$w_{\overline{W}}$	0.238	0.283		
	w_{MF}	0.344	0.409	0.376	6.5
MARKET HOGS	$w_{\mathbf{T}}$	0.169	0.194	0.180	5.2
	w _F	0.159	0.181		
	ww	0.253	0.287	0.268	
	$\mathbf{w}_{\mathbf{MF}}$	0.346	0.420	0.382	6.2
UNDER 60 LB	₩Ţ	0.173	0.195	0.182	3.8
	w r	0.156	0.182	0.172	
	$\mathbf{w}_{\mathbf{W}}^{\perp}$	0.251	0.297		
	w_{MF}	0.327	0.405	0.375	6.9
60-119 LB	w _T	0.159	0.193		
	wr	0.138	0.182	0.168	
	w _w	0.253	0.291	0.270	
	w_{MF}	0.360	0.416	0.387	5.2
120-179 LB	W۳	0.167	0.193	0.180	4.7
	$\mathbf{w}_{\mathbf{F}}^{\perp}$	0.161	0.193	0.179	5.5
	. ww	0.264	0.313	0.279	5.9
	$w_{\mathbf{MF}}^{N}$	0.326	0.408	0.362	7.6
180 LB UP	₩Ţ	0.153	0.213	0.178	12.2
	w _F	0.164	0.204	0.179	7.3
	ww	0.248	0.327	0.275	9.0
	$w_{\mathbf{MF}}$	0.262	0.418	0.368	13.9
BIRTHS	w _F	0.202	0.227	0.213	4.7
	w _w	0.302	0.360	0.328	6.2
	w_{MF}	0.416	0.496	0.459	5.8
DEATHS	w _F	0.123	0.263	0.207	23.1
	w _W	0.134	0.372	0.301	23.7
	77	0.425	0.742	0.492	21.0

TABLE C.4 SUMMARY STATISTICS FOR THE WEIGHTS OF THE S.INV.CV COMPOSITE: TOTAL HOGS FOR EIGHT-STATES BY CATEGORY

========			========		
					COEFF. OF
CATEGORY	<u>WEIGHT</u>	<u>MINIMUM</u>	<u>MAXIMUM</u>	<u>MEAN</u>	<u>VARIATION</u>
TOTAL HOGS	••	0.181	0.187	0.183	1.0
TOTAL HOGS	$^{\mathtt{W}}\mathbf{T}$	0.161	0.172	0.168	1.1
	$\mathbf{w_F}$	0.165		0.274	
		0.263	0.284		
	$w_{\mathbf{MF}}$	0.362	0.387	0.374	2.5
BREED HOGS	۷Ţ	0.184	0.194	0.189	1.8
	wF	0.172	0.176	0.174	0.9
	ww	0.257	0.285	0.274	3.8
	$w_{\mathbf{MF}}^{\mathbf{W}}$	0.349	0.383	0.363	3.6
MARKET HOGS	v. v	0.180	0.186	0.182	1.1
MARKET HOGS			0.138	0.162	
		0.166			2.7
		0.264	0.284	0.275	
	$w_{\mathbf{MF}}$	0.361	0.386	0.374	2.4
UNDER 60 LB	Wт	0.182	0.189	0.186	1.4
	$w_{\mathbf{F}}^{\perp}$	0.168	0.177	0.173	1.7
	ww	0.267	0.282	0.275	1.7
	w_{MF}	0.353	0.380	0.367	2.7
60-119 LB	$\mathbf{w}_{\mathbf{T}}$	0.172	0.180	0.176	1.5
00 113 11	w _T	0.165	0.174	0.171	1.8
		0.267	0.282	0.275	1.8
	W _W	0.372	0.391	0.379	1.7
	WMF	0.372	0.331	0.373	1.,
120-179 LB	$\mathbf{w}_{\mathbf{T}}$	0.180	0.185	0.182	1.2
	wr	0.175	0.181	0.179	1.2
	ww	0.275	0.298	0.286	2.7
	$w_{\mathbf{MF}}^{\mathbf{N}}$	0.337	0.365	0.353	3.0
180 LB UP	W —	0.173	0.190	0.180	3.2
100 LD OF	$\mathbf{w_T}$	0.174	0.185	0.179	1.8
	$^{w_{\mathbf{F}}}$				
	WW	0.270 0.334	0.291 0.378	0.279 0.362	2.6 3.7
	w_{MF}	0.334	0.376	0.362	3./
BIRTHS	${f w_F}$	0.213	0.219	0.216	1.0
	ww	0.326	0.350	0.339	2.6
	w_{MF}	0.430	0.458	0.444	2.3
DEATHS	w _F	0.187	0.228	0.211	6.3
	w _F .	0.274	0.320	0.310	5.3
	${\tt w_{MF}}$	0.454	0.528	0.478	5.8
	MF				

TABLE C.5 SUMMARY STATISTICS FOR THE WEIGHTS OF THE INV.VAR COMPOSITE: TOTAL HOGS BY STATE

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STATE	WEIGHT	MINIMUM	MAXIMUM	MEAN	COEFF. OF VARIATION
IL	$\mathbf{w_{T}}$	0.042	0.151	0.083	50.8
	w _F	0.032	0.115	0.061	46.4
	ww	0.117	0.332	0.206	29.6
	WMF	0.510	0.809	0.650	16.3
IN	W۳	0.090	0.333	0.139	59.4
	wr	0.050	0.149	0.093	32.2
	w _W	0.177	0.416	0.269	29.5
	w_{MF}	0.290	0.677	0.499	24.5
IA	$\mathbf{w_{T}}$	0.081	0.205	0.139	28.3
	$\mathbf{w_F}$	0.078	0.128	0.104	18.9
	$w_{\overline{W}}$	0.158	0.302	0.231	22.3
	w_{MF}	0.407	0.603	0.526	11.8
KA	WT	0.068	0.195	0.141	31.4
	wF	0.062	0.200	0.108	40.9
	$w_{\overline{W}}$	0.143	0.352	0.243	26.5
	WMF	0.401	0.727	0.507	20.7
MN	WT	0.073	0.199	0.134	33.6
	$\mathbf{w}_{\mathbf{F}}^{-}$	0.069	0.200	0.129	33.0
	WW	0.185	0.297	0.233	17.6
	w_{MF}	0.356	0.631	0.503	22.8
MO	Wqp	0.076	0.158	0.100	27.0
	$\mathbf{w}_{\mathbf{F}}^{-}$	0.063	0.117	0.085	23.5
	WW	0.231	0.367	0.302	16.2
	$w_{\mathbf{MF}}$	0.411	0.584	0.513	10.8
NB	$\mathbf{w_{T}}$	0.061	0.165	0.109	28.0
	$\mathbf{w}_{\mathbf{F}}^{-}$	0.059	0.131	0.099	24.6
	WW	0.148	0.273	0.222	21.6
	$w_{\mathbf{MF}}$	0.474	0.703	0.570	15.5
ОН	$\mathbf{w_{T}}$	0.131	0.249	0.190	20.9
	$\mathbf{w}_{\mathbf{F}}^{-}$	0.074	0.212	0.127	38.0
	WW	0.118	0.340	0.235	28.3
	w _{MF}	0.382	0.526	0.448	12.3

TABLE C.6 SUMMARY STATISTICS FOR THE WEIGHTS OF THE S.INV.VAR COMPOSITE: TOTAL HOGS BY STATE

=====				:=======	
STATE	WEIGHT	MUMINIM	MUMIXAM	MEAN	COEFF. OF VARIATION
IL	$\mathbf{w_{T}}$	0.073	0.120	0.096	17.7
	WF	0.052	0.079	0.062	14.4
	$\mathbf{w}_{\mathbf{W}}^{-}$	0.198	0.273	0.226	12.6
	w_{MF}	0.555	0.678	0.615	8.1
IN	w _T	0.109	0.165	0.126	14.7
	$\mathbf{w}_{\mathbf{F}}^{\perp}$	0.079	0.098	0.087	8.2
	$\mathbf{w}_{\mathbf{W}}^{-}$	0.247	0.349	0.295	12.4
	$\mathbf{w_{MF}}$	0.444	0.557	0.492	7.5
IA	$\mathbf{w_{T}}$	0.109	0.150	0.128	13.2
	$\mathbf{w}_{\mathbf{F}}^{-}$	0.092	0.110	0.099	7.1
	w _w	0.211	0.276	0.253	9.3
	$\mathbf{w}_{\mathbf{MF}}$	0.496	0.532	0.520	2.3
KA	W _T	0.125	0.153	0.139	6.8
	$\mathbf{w}_{\mathbf{F}}^{-}$	0.099	0.125	0.108	9.2
	$\mathbf{w}_{\mathbf{W}}^{-}$	0.228	0.296	0.262	10.2
	w_{MF}	0.466	0.533	0.491	4.8
MN	WT	0.121	0.171	0.149	13.4
	$\mathbf{w}_{\mathbf{F}}^{-}$	0.114	0.160	0.140	12.1
	ww	0.222	0.258	0.242	5.2
	w_{MF}	0.418	0.543	0.469	9.6
MO	₩ Ţ	0.101	0.151	0.117	14.7
	wr	0.084	0.113	0.093	10.7
	$w_{\overline{W}}$	0.285	0.337	0.319	5.3
	w_{MF}	0.408	0.521	0.471	8.0
NB	$\mathbf{w_T}$	0.104	0.137	0.117	10.5
	$\mathbf{w_F^-}$	0.092	0.107	0.101	5.3
	www.	0.186	0.229	0.204	6.7
	w_{MF}	0.561	0.618	0.579	3.7
ОН	$\mathbf{w_{T}}$	0.155	0.195	0.181	6.9
	$\mathbf{w_F^2}$	0.089	0.144	0.112	18.9
	ww	0.221	0.325	0.272	12.7
	WMF	0.421	0.456	0.435	3.2

TABLE C.7 SUMMARY STATISTICS FOR THE WEIGHTS OF THE INV.CV COMPOSITE: TOTAL HOGS BY STATE

2=2==	. * * * * * * * * * * * * * * * * * * *	CV COMPOS.	ITE: TOTAL	HOGS BI	STATE ======
STATE	WEIGHT	MINIMUM	MAXIMUM	<u>MEAN</u>	COEFF. O
211111			***********	**************************************	
IL	$\mathbf{w_{T}}$	0.127	0.194	0.158	15.7
	$\mathbf{w}_{\mathbf{F}}^{-}$	0.126	0.182	0.151	12.1
	WW	0.219	0.297	0.250	
	w_{MF}	0.367	0.522	0.442	12.5
IN	Wm	0.144	0.249	0.166	20.9
	$\mathbf{w}_{\mathbf{F}}^{\perp}$	0.128	0.185	0.154	11.4
	$w_{\overline{W}}$	0.229	0.333	0.272	12.1
	$w_{\mathbf{MF}}$	0.303	0.492	0.408	13.9
IA	w _T	0.164	0.215	0.189	8.7
	wr	0.158	0.188	0.172	5.7
	w _w	0.236	0.279	0.261	6.3
	$w_{MF}^{"}$	0.326	0.410	0.378	7.3
KA	$w_{\mathbf{T}}$	0.161	0.228	0.200	12.8
	w _F	0.148	0.231	0.191	14.2
	w _w	0.210	0.275	0.256	8.4
	$w_{MF}^{"}$	0.282	0.458	0.353	16.7
MN	w _T	0.167	0.209	0.193	6.8
	wr	0.174	0.214	0.191	7.2
	ww	0.247	0.297	0.269	6.3
	w_{MF}^{N}	0.302	0.388	0.348	10.1
MO	Wη	0.152	0.187	0.163	6.7
	$\mathbf{w_F^1}$	0.143	0.168	0.156	5.6
	w _w	0.260	0.324	0.298	8.1
	w_{MF}	0.357	0.420	0.384	6.2
NB	٧ _T	0.164	0.210	0.190	8.4
	$\mathbf{w}_{\mathbf{F}}^{1}$	0.151	0.196	0.177	8.3
	M _r	0.222	0.281	0.261	7.1
	w_{MF}^{N}	0.314	0.430	0.373	10.8
ОН	$\mathbf{w_{T}}$	0.155	0.212	0.185	8.7
	$\mathbf{w_F}$	0.157	0.176	0.168	4.0
	w _W	0.232	0.306	0.270	8.0
	$\mathbf{w_{MF}}$	0.347	0.429	0.378	7.9

TABLE C.8 SUMMARY STATISTICS FOR THE WEIGHTS OF THE S.INV.CV COMPOSITE: TOTAL HOGS BY STATE

	9.IN	V.CV COMPO	JSITE: TOT	AL HUGS	BI STATE
STATE	WEIGHT	MINIMUM	MAXIMUM	MEAN	COEFF. OF VARIATION
IL	w _T	0.151	0.182	0.167	7.3
	٧̈́F	0.144	0.159	0.150	3.7
	w _W	0.247	0.277	0.258	
	w_{MF}	0.395	0.455	0.425	6.1
IN	$w_{\mathbf{T}}$	0.155	0.178	0.163	4.9
		0.147	0.157	0.152	2.1
	$\mathbf{w}_{\mathbf{W}}^{-}$	0.264	0.309	0.286	6.1
	WMF	0.376	0.434	0.399	4.9
IA	$w_{\mathbf{T}}$	0.180	0.194	0.188	2.7
	wF	0.170	0.176	0.173	1.0
	WW	0.256	0.279	0.270	3.4
	WMF	0.357	0.381	0.370	1.8
KA	w _T	0.190	0.207	0.197	3.3
	$\mathbf{w}_{\mathbf{F}}^{-}$	0.179	0.198	0.189	3.8
	WW	0.251	0.270	0.261	2.8
	$w_{\mathbf{MF}}$	0.339	0.369	0.353	3.3
MN	${f w_T}$	0.190	0.201	0.196	2.5
	$w_{\mathbf{F}}$	0.186	0.198	0.192	2.2
	WW	0.265	0.277	0.271	1.7
	$w_{\mathbf{MF}}$	0.326	0.359	0.340	3.4
MO	${f w_T}$	0.163	0.185	0.171	4.4
	$w_{\mathbf{F}}^-$	0.156	0.170	0.160	2.9
	w_W	0.289	0.310	0.301	2.2
•	$w_{\mathbf{MF}}$	0.349	0.388	0.368	3.4
NB	${f w_T}$	0.187	0.203	0.195	3.3
	$\mathbf{w}_{\mathbf{F}}^{-}$	0.175	0.185	0.179	1.8
	w_W	0,251	0.262	0.257	1.4
	w_{MF}	0.350	0.384	0.369	3.0
ОН	${f w_T}$	0.178	0.188	0.184	2.1
	$\mathbf{w}_{\mathbf{F}}$	0.167	0.171	0.170	0.8
	w_W	0.266	0.297	0.280	3.8
	$w_{\mathbf{MF}}$	0.353	0.389	0.365	3.3

APPENDIX D

SUMMARY TABLES FOR THE MULTIVARIATE ANALYSES OF BIASES

The following ten tables present MANOVAs conducted to determine which composites were statistically different from each other using Tukey multiple comparisons with $\alpha=0.05$. They are divided into two sections. The first section of eight tables shows which composites are significantly different for individual states. The second section of two tables presents the comparison between the eight year ASB estimates and the means for each category (total hogs, breed, and so forth) according to category and then state.

All tables that present information on the significantly different composites use a one way ordering of the most different to the least different composites for each of the eight hog categories. That is, when the m.frame is different from a group of composites, then each of those composites are different from the m.frame as well. example, the line labeled m.frame of table D.1.1 shows that for Illinois that composite 7 (the multiple frame) is most significantly different from composite 4 (midrange), then composite 1 (equal), next composite 6 (smoothed inverse variance), and finally composite 3 (inverse variance) for the total hogs category. The significance level for the test of hypothesis that all the composites are the same for the total category is 0.0001; that is, the composites are highly unlikely to have the same underlying means for the total hogs category. When no number appears for a category and composite, the composites are not significantly different at the p value (Pr > F line) below the category title.

The last two tables list the counts of the number of times a composite mean is closest to a category mean for that composite. The tables present the counts according to item codes and the composites.

Tables D.1.1 - D.1.8 MANOVA Analysis of Individual States

Tables D.1.1 through D.1.8 present the significantly different composites on a state-by-state basis. The tables show that there is a great degree of variability among the states about which composites are significantly different for each category.

Missouri (Table D.1.6) is the only state for which the composites are not significantly different in any hog category. Iowa (Table D.1.3) has only three categories (Total, Breed, and Market) for which composites are significantly different. Minnesota (Table D.1.5), Kansas(Table D.1.4) and Ohio (Table D.1.8) have from five to all categories for which the multiple frame composite is statistically different from the other composites. Illinois (Table D.1.1) and Nebraska (Table D.1.7) have the largest number of composites statistically different.

As a result, these tables show that there is a great degree of variability among the states about whether the various composites are statistically different from each other. This variability leads to the question of which composite most closely approximates the ASB average for the eight year period for each state. The last two tables answer this question.

Tables D.2.1 - D.2.2 MANOVA Analysis of Individual States: Tables for Comparison of Composites

Tables D.2.1 and D.2.2 present the number of times that the eight hog category means for the composites were closest to the board eight year average when considering both the individual hog categories (Table D.2.1) and the states (Table D.2.2). This is one way to compare the general trends of the composites to the board estimates.

Table D.2.1 shows no category has more than five ASB means closest to the multiple frame for any category except the 120-179 pound category. However, the multiple frame occurs most often among the categories with 24 occurrences while the mid.range is the second most frequent with 12 occurrences. The remaining composites occur with nearly equal frequencies among the categories.

Table D.2.2 shows that the smoothed inverse variance composite occurs for six hog categories for Indiana, the mid.range occurs six times for Iowa and the multiple frame occurs five times for Ohio. The remaining states do not have a strong pattern.

D.1 MANOVA ANALYSES OF INDIVIDUAL STATES

D.1.1 SIGNIFICANTLY DIFFERENT COMPOSITES FOR AVERAGE BIAS FOR 1979 - 1986

STATE: ILLINOIS

			1	HOG AND	SIG IME	Ā		
COMPOSITE	TOTAL	BRIDED	MARKET	UNDER60	60-119	120-179	180UP	BIRTHS
equal inv.var				41		4		41
inv.cv mid.range				4		4		41
s.inv.var s.inv.cv mult.frame	4	4				41		416
PR > F					.1432	.0001	.3220	
	Note 1: The composite abbreviations are defined on pages 4 and 5.							
	ll the	compos:	ites an	e the sa	me for	for reject that hog the ord	item.	
				st diffe		ule ord	er m	WIIICII

D.1.2 SIGNIFICANTLY DIFFERENT COMPOSITES FOR AVERAGE BIAS FOR 1979 - 1986

STATE: INDIANA

	_	HOG AND			1000	Paromio
COMPOSITE TOTAL BREET	MARKET	UNDER60	<u>60-119</u>	120-179	TROOP	BIRIHS
equal inv.var inv.cv mid.range	41		4			
s.inv.var 41 s.inv.cv	41	14	14 4			
mult.frame 52634 26314	526341	523614	562314	562314	52634	1 26314
PR > F .0001 .0001	.0001	.0001	.0001	.0001	.0001	.0001
Note 1: See notes 1 t	hrough	3 for Tal	ble D.1	.1.		

D.1.3 SIGNIFICANTLY DIFFERENT COMPOSITES FOR AVERAGE BIAS FOR 1979 - 1986

STATE: IOWA

]	HOG AND	PIG ITE	M		
COMPOSITE	TOTAL	BRIDID	MARKET	UNDER60	60-119	<u>120-179</u>	180UP	BIRIHS
equal inv.var inv.cv mid.range s.inv.var s.inv.cv mult.frame	a <i>4</i> 1	4163	4					
Pr > F				.3335	.9560	.9593	.9873	.6205
Note 1:	See not	es 1 t	hrough	3 for Tal	ble D.1	.1.		

D.1.4 SIGNIFICANTLY DIFFERENT COMPOSITES FOR AVERAGE BIAS FOR 1979 - 1986

STATE: KANEAS

		HOG AND PIG ITE	1	
COMPOSITE TOTAL	BREED MARKET	<u>UNDER60</u> 60-119	120-179 180UP	BIRTHS
equal inv.var inv.cv mid.range s.inv.var s.inv.cv	4	14		
mult.frame 4136	4136	41 14635	143	
PR > F .0001	.3787 .0001	.0002 .0001	.0558 .0053	.7274
Note 1: See not	tes 1 through	3 for Table D.1.	1.	

D.1.5 SIGNIFICANTLY DIFFERENT COMPOSITES FOR AVERAGE BIAS FOR 1979 - 1986

STATE: MINNESOTA

HOG AND PIG ITEM

COMPOSITE TOTAL BREED MARKET UNDERGO 60-119 120-179 180UP BIRTHS

equal

inv.var

inv.cv

mid.range

s.inv.var

s.inv.cv

mult.frame 16435 16 16435 1463 163452 163

PR > F .0001 .0208 .0001 .0716 .0003 .0001 .1200 .0085

Note 1: See notes 1 through 3 for Table D.1.1.

D.1.6 SIGNIFICANTLY DIFFERENT COMPOSITES FOR AVERAGE BIAS FOR 1979 - 1986

STATE: MISSOURI

HOG AND PIG ITEM

COMPOSITE TOTAL BREED MARKET UNDERGO 60-119 120-179 180UP BIRTHS

equal

inv.var

inv.cv

mid.range

s.inv.var

s.inv.cv

mult.frame

PR > F 1.0000 .5302 .9968 .9976 .9799 .9925 .5243 .9919

Note 1: See notes 1 through 3 for Table D.1.1.

D.1.7 SIGNIFICANTLY DIFFERENT COMPOSITES FOR AVERAGE BIAS FOR 1979 - 1986

STATE: NEBRASKA

]	HOG AND	PIG ITE	Y		
COMPOSITE	TOTAL	BREED MARKET	UNDER60	60-119	120-179	180UP	BIRIHS
equal inv.var inv.cv	1463	16	1463	1	1	1463	14
mid.range s.inv.var s.inv.cv	1463	1	1463		1	14	1
mult.frame	146352	(FOR ALL CAT	EGORIES)				
PR > F .0	0001	.0001 .0001	.0001	.0001	.0001	.0001	.0001
Note 1: S	See not	es 1 through :	3 for Tal	ole D.1	.1.		

D.1.8 SIGNIFICANTLY DIFFERENT COMPOSITES FOR AVERAGE BIAS FOR 1979 - 1986

STATE: OHIO

American de la Companya de la Compa			H	og and	PIG ITE	1		
COMPOSITE	TOTAL	BREED N	ARKET	UNDER60	60-119	120-179	180UP	BIRIHS
equal inv.var inv.cv mid.range s.inv.var								
s.inv.cv mult.frame	=	41 412635	412365	412653	24	2	41253	416
PR > F	.0001	0001	.0001	.0001	.0362	.0238	.0001	.0001
Note 1: S	See note	s 1 thr	rough 3	for Ta	ble D.1	1.		

D.2 TABLES FOR COMPARISON OF COMPOSITES

D.2.1 NUMBER OF STATES WITH LEAST AVERAGE BIAS FOR EACH ITEM

CATEGORY		-	9	COMPOSIT	E		
	1	2	3	4	<u>5</u>	<u>6</u>	7
TOTAL	0	1	0	1	1	1	4
BREED	0	1	1	1	2	1	2
MARKET	0	1	0	1	1	1	4
UNDER60	1	0	1	3	0	1	2
60-119	0	4	0	0	0	0	4
120-179	0	1	0	1	1	0	5
180UP	3	1	0	2	0	0	2
BIRIHS	1	0	2	3	1	0	1
TOTAL	5	9	4	12	6	4	24

D.2.2 NUMBER OF ITEMS WITH LEAST AVERAGE BIAS FOR EACH STATE

· · · · · · · · · · · · · · · · · · ·							
STATE				OMPOSIT	E		
	1	2	3	4	5	<u>6</u>	7
ILLINOIS	1	1	0	2	0	3	1
INDIANA	0	0	0	0	2	0	6
IOWA	0	1	0	6	0	0	1
KANSAS	0	3	0	2	1	0	2
MINNESOTA	2	1	0	1	0	0	4
MISSOURI	0	1	1	1	0	1	4
NEBRASKA	1	1	2	0	3	0	1
онто	1	1	1	0	0	0	5
TOTAL	5	9	4	12	6	4	24

APPENDIX E

SUMMARY TABLES FOR THE NONPARAMETRIC ANALYSES FOR FOUR EVALUATION CRITERIA

The tables in this section present the data analyses using nonparametric methods of analyzing ranks in the form of MANOVA, Univariate ANOVA, and Tukey's Multiple Comparisons (with 95 per cent confidence) to establish relative differences among the composites. There are five overall tables, each of which has five or six subtables. The last table has a sixth part which explains the comparisons in terms of a total mean for each composite for all items and all states.

Table E.1 (1.1-1.4) contains the mean ranks over the eight hog categories and significant differences among the composites for the four evaluation criteria (absolutes bias, absolute difference, root mean square error, and standard deviation) for each of the two methods of computing a eight-state composite. The Table E.1 (1.5-1.6) also contains several multivariate tests for the significance of the four criteria and an analysis treating all four criteria equally. Using either method of computing a eight-state composite, the four criteria taken together suggest that the mid.range composite comes closest to the board (see Tables E.1.5 and E.1.6).

Tables E.2 (2.1-2.5) and E.3 (3.1-3.5) summarize, by state and by evaluation criteria, analyses over hog categories which compare mean ranks and determine significant differences. Tables E.2.5 and E.3.5 show that for equal treatment of the four criteria the s.inv.var and the inv.var are closest to the board for five of the eight states. Table E.3.5 also shows that, for equal treatment of the four evaluation criteria, the composites are often different from one another.

Tables E.4 and E.5 present the same information about the various hog item codes (total, market, breed, under60, 60-119, 120-179, 180up, births, and an average of these categories) as Tables E.2 and E.3 did for the states. Finally, Table E.6 shows that, on the average for the four criteria, the smoothed inverse variance composite most closely follows the board for the previously listed hog categories.

E.1 RANKS AND SIGNIFICANT DIFFERENCES AMONG THE COMPOSITES FOR THE GIVEN EVALUATION CRITERION

		of State Osites	COMPOS STATE	SITE OF SUMS
COMPOSITE	MEAN RANK	DIFFERENT	MEAN RANK	DIFFEREN
equal	3.3		3.0	
inv.var	5.1	4	5.0	4
inv.cv	4.1		4.0	
mid.range	1.9**	72	2.5 **	
s.inv.var	4.5		4.5	
s.inv.cv	3.8		3.5	
mult.frame	5.4	4	5.5	4

	SUM OI COMPOS	F STATE SITES	COMPO: STATE	SITE OF SUMS
COMPOSITE	MEAN RANK	DIFFERENT	MEAN RANK	DIFFERENT
equal	3.3		3.5	
inv.var	5.1	4	5.0	
inv.cv	4.0		3.6	
mid.range	2.1 **	72	2.4 **	7
s.inv.var	4.5		4.5	
s.inv.cv	3.6		3.5	
mult.frame	5.4	4	5.5	4

	SUM OI COMPO	f State Sites	COMPO	SITE OF SUMS
COMPOSITE	MEAN RANK	DIOGRAMM	MEAN RANK	DIGGORIONI
equal	3.5	He same see	3.8	
inv.var	5.1	2	4.9	
inv.cv	4.1		3.6	
mid.range	2.0 **	72	2.9 **	
s.inv.var	4.6		4.0	
s.inv.cv	3.4		3.5	
mult.frame	5.3	4	5.4	

	SUM OF COMPOS	state Sites	COMPOS STATE	SITE OF SUMS
COMPOSITE	MEAN RANK	DIFFERENT	MEAN RANK	DIFFERENT
equal	6.5	2573	6.3	2573
inv.var	2.4 **	14	1.6 **	1463
inv.cv	3.8	1	3.8	142
mid.range	4.9	2	6.0	2573
s.inv.var	3.0	1	2.6	146
s.inv.cv	4.3		4.7	25
mult.frame	3.2	1	3.0	14

E.1.5 MULTIVARIATE ANALYSIS OF VARIANCE FOR ABSOLUTE BIAS, ABSOLUTE DIFFERENCE, ROOT MEAN SQUARE ERROR, AND STANDARD DEVIATION

MANOVA TEST	F	<u>Pr></u> F
Wilks'	0.0003	0.0001
Pillai's	0.0020	0.0001
Hotelling's	0.0001	0.0001
Roy's	0.0001	0.0001

E.1.6 EQUAL TREATMENT OF THE ABSOLUTE BIAS, ABSOLUTE DIFFERENCE, ROOT MEAN SQUARE ERROR, AND STANDARD DEVIATION

	SUM OI COMPOS	F STATE SITES	COMPOSITE OF STATE SUMS				
COMPOSITE	MEAN RANK	DIPPERENT	MEAN RANK	DIFFERENT			
equal inv.var inv.cv mid.range s.inv.var s.inv.cv mult.frame	4.1 4.4 4.0 2.7 ** 4.2 3.7 4.8	4 72 4	4.1 4.1 3.8 3.4 ** 3.9 3.8 4.8				
ANOVA Pr >	F	0.002		0.2			

Note 1: These tables summarize Multivariate Analysis of Variance, Univariate Analysis of Variance, and Tukey's Multiple Comparisons with 95 percent confidence on pairwise comparisons.

E.2 BY STATE SUMMARIES OF THE MEAN RANKS OF THE COMPOSITES FOR THE GIVEN EVALUATION CRITERION

E.2.1 ABS	OLUTE	BIAS										
		STATE										
COMPOSITE	щ	IN	IA	KA	MN	MO	NB	OH				
equal	3.4	6.2	3.3	5.1	5.1	4.9	5.3	5.7				
inv.var	4.5	3.1	4.9	3.0*	3.4*	5.1	3.4	5.2				
inv.cv	3.1	4.6	4.1	3.9	3.8	3.4	2.9	3.2				
mid.range	4.8	6.6	1.6*	5.3	3.9	5.5	4.3	6.6				
s.inv.var	3.9	1.8*	4.4	3.1	3.7	2.7*	2.8	2.6				
s.inv.cv	2.7	3.5	4.0	3.6	4.5	3.1	3.8	3.0				
mult.frame	5.8	2.3	5.6	3.9	3.6	3.4	5.8	1.8*				
ANOVA Pr>F	0.02	0.0001	0.001	0.2	0.6	0.01	0.009	0.0001				

Note 1: Estimates are averages for 1979-1986 data, and the tests use each of the seven items as an observation.

	STATE									
COMPOSITE	IL	<u>IN</u>	<u>IA</u>	<u>KA</u>	MN	<u>M</u> O	NB	<u>OH</u>		
equal	4.2	6.3	4.9	5.8	6.4	6.7	5.6	5.7		
inv.var	3.4	2.7	4.1	3.4	2.3*	2.7	2.2	4.6		
inv.cv	2.9	4.3	3.3	4.4	3.9	4.3	3.8	2.7		
mid.range	5.9	6.7	3.9	5.9	5.3	6.1	4.4	6.6		
s.inv.var	2.8*	2.1	3.4	2.4	2.7	2.3	2.1*	2.5		
s.inv.cv	3.2	3.9	3.1*	4.0	4.6	4.1	4.1	3.1		
mult.frame	5.8	2.0*	5.4	2.1*	2.9	1.9*	5.8	2.8		

E.2.3 ROC	yt mean	SQUARE	ERROR					
				STA	TE			
COMPOSITE	ш	IN	IA	KA	MN	MO	NB	⊘ H
equal	4.9	6.3	4.8	5.8	6.5	6.7	5.6	5.8
inv.var	3.9	3.2	4.2	3.3	2.4*	2.6	2.0*	4.3
inv.cv	2.6	4.1	3.4	4.4	3.8	4.1	3.9	2.8
mid.range	6.3	6.6	4.1	5.9	5.1	6.3	4.4	6.8
s.inv.var	2.4*	2.0	3.1*	2.7	2.4*	2.1	2.3	2.6
s.inv.cv	2.7	4.0	3.1	3.9	5.0	4.3	4.1	3.3
mult.frame	5.3	1.9*	5.4	2.0*	2.9	1.9*	5.8	2.4*
ANOVA Pr>F	0.0001	0.0001	0.2	0.0001	0.0001	0.0001	0.0001	0.0001

Note 1: Estimates are averages for 1979-1986 data, and the tests use each of the seven items as an observation.

E.2.4 STA	E.2.4 STANDARD DEVIATION											
	STATE											
COMPOSITE	IL	IN	<u>IA</u>	KA	MN	MO	NB	ŒH.				
equal 'inv.var inv.cv mid.range s.inv.var s.inv.cv mult.frame	5.9 2.1 4.1 6.9 1.8* 3.9 3.3	5.4 4.7 2.9 5.4 3.7 4.2 1.8*	5.6 2.3* 3.8 5.7 2.9 4.6 3.3	6.1 2.9 5.1 6.0 2.3 4.1 1.6*	6.9 2.0* 4.3 5.3 2.4 4.8 2.4	6.3 1.9* 4.2 5.6 2.9 4.4 2.8	6.5 2.0 4.6 4.3 1.9* 4.1 4.8	5.8 3.6 2.4* 5.4 2.9 3.6 4.4				
ANOVA Pr>F	0.0001	0.0004	0.0006	0.0001	0.0001	0.0001	0.0001	0.001				

E.2.5 EQUAL TREATMENT OF ABSOLUTE BIAS, ABSOLUTE DIFFERENCE, ROOT MEAN SQUARE ERROR, AND STANDARD DEVIATION

		STVAVE										
COMPOSITE	IL	IN	<u>IA</u>	<u>KA</u>	W	MO	NB	⊘H				
equal	4.6	6.0	4.6	5.7	6.2	6.1	5.8	5.8				
inv.var	3.5	3.4	3.9	3.2	2.5*	3.1	2.4	4.4				
inv.cv	3.2	4.0	3.7	4.5	3.9	4.0	3.8	2.8				
mid.range	5.9	6.3	3.8	5.8	4.9	5.9	4.4	6.4				
s.inv.var	2.7*	2.4	3.4*	2.6	2.8	2.5	2.2*	2.7*				
s.inv.cv	3.1	3.9	3.7	3.9	4.7	4.0	4.0	3.2				
mult.frame	5.0	2.0*	4.9	2.4*	3.0	2.5*	5.5	2.8				
ANOVA Pr>F	0.0001	0.0001	0.02	0.0001	0.0001	0.0001	0.0001	0.0001				

Note 1: Estimates are averages for 1979-1986 data, and the tests use each of the seven items as an observation.

E.3 BY STATE SUMMARIES OF THE SIGNIFICANT DIFFERENCES AMONG THE THE COMPOSITES FOR THE GIVEN EVALUATION CRITERION

E.3.1 ABSOLUTE BIAS											
	•			SI	ATE						
COMPOSITE	IL	ĪN	<u>la</u>	<u>ka</u>	MN	<u>M</u> O	<u>NB</u>	<u>OH</u>			
equal.		5726						7563			
inv.var		41	4					7563			
inv.cv		574					3	412			
mid.range		57263	725			5		7563			
s.inv.var		4136	4			4	7	412			
s.inv.cv	7	415						412			
mult.frame	6	413	4				53	412			
ANOVA Pr>F	0.02	0.0001	0.001	0.2	0.6	0.01	0.009	0.000			

E.3.2 AB9												
		STATE										
COMPOSITE	IL	IN	IA	KA	WN	MO	NB	Œ				
equal	5	75263		752	2573	75263	52	5376				
inv.var	4	413		41	146	14	71	5				
inv.cv	47	47512		7	1	1754		41				
mid.range	5362	75263		752	25	75263	;	5376				
s.inv.var	47	4136		41	14	1436	71	412				
s.inv.cv	4	4175			2	1745		41				
mult.frame	53	4136		413	1	1437	52	41				
ANOVA Pr>F	0.0005	0.0001	0.2	0.0001	0.0001	0.0001	0.0001	0.0001				

Note 1: Estimates are averages for 1979-1986 data, and the tests use each of the seven items as an observation.

E.3.3 ROO	r mean (equare e	RROR					
				<u>517</u>	<u>TE</u>			
COMPOSITE	IL	<u>IN</u>	<u>IA</u>	<u>KA</u>	MN	<u>MO</u>	<u>NB</u>	<u>OH</u>
equal	5	75263		752	5273	75236	25	7536
inv.var	4	41		41	146	1463	714	47
inv.cv	47	4175		7	1	17452		41
mid.range	5362	75263		752	52	75236	2	75362
s.inv.var	471	4136		41	146	1463	71	41
s.inv.cv	47	4175			52	71542		41
mult.frame	536	4136		413	1	1463	25	412
ANOVA Pr>F	0.0001	0.0001	0.2	0.0001	0.0001	0.0001	0.0001	0.0001

T 1		DEVIATION
F - 1 - A	PET AND MARKET	

	STATE								
COMPOSITE	IL	IN	IA	KA	MN	MO	NB	<u>OH</u>	
equal	52763	7	25	7526	27536	275	5264	35	
inv.var	4136	7	41	143	1463	1463	1734		
inv.cv	4521			752	1275	2	52	14	
mid.range	52763	7	25	7526	275	275	521	3	
s.inv.var	4136		41	1436	1463	14	17346	1	
s.inv.cv	4512			7154	2751	2	15		
mult.frame	41	142		1436	1463	14	52		

ANOVA Pr>F 0.0001 0.0004 0.0006 0.0001 0.0001 0.0001 0.0001 0.001

Note 1: Estimates are averages for 1979-1986 data, and the tests use each of the seven items as an observation.

E.3.5 EQUAL TREATMENT OF ABSOLUTE BIAS, ABSOLUTE DIFFERENCE, ROOT MEAN SQUARE ERROR, AND STANDARD DEVIATION

STATE

COMPOSITE	IL	<u>IN</u>	<u>IA</u>	<u>KA</u>	MN	MO	<u>NB</u>	<u>OH</u>
equal inv.var inv.cv mid.range s.inv.var s.inv.cv mult.frame	5634 47 471 56321 471 471 5631	75263 4175 4175 75263 41362 4175 41362	7 5	75263 413 75421 75263 4136 4175 4136	257364 1463 12 2571 146 2571 146	75263 14 1475 75263 1436 1475 1436	52364 17463 1752 5217 17463 5127 5236	53762 453176 412 53762 412 412 412

ANOVA Pr>F 0.0001 0.0001 0.02 0.0001 0.0001 0.0001 0.0001 0.0001

E.4 BY HOG AND PIG ITEM SUMMARIES OF THE MEAN RANKS OF THE COMPOSITES FOR THE GIVEN EVALUATION CRITERION

E.4.1 AB	BOLUTE	BIAS							
COMPOSITE	TOTAL	ereed M	ARKET	UNDER60	<u>60-119</u>	120-179	180UP	BIRTHS	AVERAC
equal	5.2	5.0	5.5	3.8	6.3	5.5	4.4	3.3	4.9
inv.var	3.8	4.0	3.6	5.6	2.4	2.9**	4.8	5.4	4.1
inv.cv	3.8	3.2	3.8	2.6**	4.2	4.3	4.1	2.9**	3.6
mid.range	5.5	4.9	5.5	3.8	6.2	4.9	4.1	3.6	4.8
s.inv.var	2.6**	2.6**	2.5*	* 4.0	2.4	3.1	3.5	4.2	3.1**
s.inv.cv	3.4	3.4	3.6	3.0	4.4	4.2	2.8**	3.2	3.5
mult.frame	3.6	4.9	3.5	5.2	1.9*	3.1	4.4	5.2	4.0

E.4.2 ABS	OLUTE !	DIFFERE	OCE						
COMPOSITE 1	OTAL	BREED M	ARKET I	UNDER60	<u>60-119</u>	<u>120-179</u>	180UP	BIRIHS	AVERAGE
equal	6.3	5.6	6.4	4.8	6.0	6.4	5.2	4.7	5.7
inv.var	2.9	2.9	2.8	4.3	2.7	1.8**	4.3	3.6	3.2
inv.cv	3.9	3.6	4.1	3.1	3.8	3.9	4.2	3.1**	3.7
mid.range	6.2	5.6	6.2	4.9	6.2	6.0	4.6	5.1	5.6
s.inv.var	2.0**	2.5**	1.6*	* 2.8**	2.4**	2.6	2.9**	3.3	2.5**
s.inv.cv	3.6	3.9	3.9	3.1	4.1	4.9	3.2	3.2	3.8
mult.frame	3.1	3.8	3.0	5.1	2.8	2.4	3.5	4.9	3.6

E.4.3 RO	E.4.3 ROOT MEAN SQUARE ERROR								
COMPOSITE	TOTAL	BREED M	ARKET	UNDER60	60-119	120-179	180UP	BIRIHS	AVERAGE
equal	6.2	5.7	6.2	5.2	6.3	6.4	5.4	4.8	5.8
inv.var	3.1	3.1	3.1	3.6	2.6	1.7**	4.8	3.9	3.2
inv.cv	3.6	3.3	3.9	3.4	3.8	4.1	4.0	3.1**	3.6
mid.range	6.1	5.4	6.1	5.3	6.4	6.1	4.9	5.2	5.7
s.inv.var	2.1**	2.7**	1.8*	* 2.1*	* 2.4*	2.5	2.6**	3.3	2.4**
s.inv.cv	3.9	3.9	3.8	3.4	3.8	4.8	3.1	3.8	3.8
mult.frame	3.0	3.9	3.1	4.9	2.7	2.5	3.3	4.0	3.4

E.4.4 STA	NDARD	DEVIATIO	ON .						
COMPOSITE T	OTAL	BREED M	ARKET U	INDER60	<u>60–119</u>	<u>120-179</u>	180UP	BIRIHS	AVERAGE
equal	6.3	5.6	6.5	6.3	5.8	6.4	5.1	6.4	6.0
inv.var	2.5**	2.9**	2.3**	2.2	3.4	1.7**	4.2	2.2**	2.7
inv.cv	3.4	3.4	3.6	4.3	3.8	3.9	4.9	3.9	3.9
mid.range	5.9	5.8	5.6	6.4	5.2	6.1	3.4	6.2	5.6
s.inv.var	2.5**	3.2	2.4	2.0**	2.4**	2.2	3.4	2.6	2.6**
s.inv.cv	4.5	3.8	4.8	4.2	3.7	4.2	4.2	4.3	4.2
mult.frame	2.9	3.4	2.9	2.6	3.6	3.6	2.8**	2.3	3.0

					-	DEVIATIO		CE,	
COMPOSITE T	OTAL	BREED M	ARKET U	NDER60	<u>60-119</u>	<u>120-179</u>	180UP	<u>BIRIHS</u>	AVERAGE
inv.var inv.cv mid.range	2.3** 3.9	3.4 5.4 2.7**	5.9 2.1**	3.4 5.1 2.7**	2.8 3.9 6.0	4.0 5.8 2.6	4.3 4.2 3.1**	3.3 5.0 3.4** 3.6	3.7 5.4 2.7

E.5 BY HOG AND PIG ITEM SUMMARIES OF THE SIGNIFICANT DIFFERENCES AMONG THE COMPOSITES FOR THE GIVEN EVALUATION CRITERION

s.5.1 Arsolut	rp privo					
COMPOSITE TOTAL	L BREED MARKE	T UNDER60	<u>60-119</u>	120-179	180UP	BURIH
equal	5		23567			
inv.var		3	346			
inv.cv			457			
nid.range	5		567			
s.inv.var			6			
s.inv.cv			7			
mult.frame						
ANOVA $Pr > F 0$.05 0.06 0.0	2 0.02	0.0001	0.05	0.6	0.06

E.5.2 A	SOLUTE	DIFFE	RENCE					
COMPOSITE	TOTAL	BREED	MARKET	UNDER60	60-119	120-179	180UP	BIRTHS
equal inv.var inv.cv mid.range s.inv.var s.inv.cv mult.frame	4 567		4 45		2357 4 4 567	23567 346 4 57 6		
ANOVA Pr	0001					0.0001	0.2	0.2

E.5.3 R	cot mea	n squa	RE ERRO	R				
COMPOSITE	TOTAL	BREED	MARKET	UNDER60	60-119	120-179	180UP	BIRIHS
equal inv.var inv.cv mid.range s.inv.var s.inv.cv mult.fram	4 4 57		23567 4 4 567	5 5 7	23567 4 4 567	346 4		
ANOVA Pr	_	0.01	0.0001	0.005	0.0001	0.0001	0.03	0.3
MANOVA Pr	> F 0	.0001,	0.0001	0.0001,	and 0.0	0001		

E.5.4 8	IANDARD	DEVIA	TION					
COMPOSITE	TOTAL	BREED	MARKET	UNDER60	60-119	120-179	180UP	BIRTH
equal	2357		2357	23567	5	23567		23567
inv.var	4	4	46	346		3467		46
inv.cv	4			457		4		4
mid.range	57		57	567	5	57		567
s.inv.var			6	6		6		
s.inv.cv								7
mult.fram	e							
			-					
ANOVA Pr	_	.006	0.0001	0.0001	800.0	0.0001	0.2	0.0001
MANOVA Pr	> F 0	.0001.	0.0009	0.0001	and 0.	.0001		

8.5.5 EQUAL TREATMENT OF ABSOLUTE BIAS, ABSOLUTE DIFFERENCE, ROOT MEAN SQUARE ERROR, AND STANDARD DEVIATION

COMPOSITE TOTAL BREED MARKET UNDER60 60-119 120-179 180UP BIRTHS

equal	23567	5	23567	5	23567	23567
inv.var	4		4		4	346
inv.cv	4		4		4	4
mid.range	567	5	57	5	567	57
s.inv.var			6			6
s.inv.cv	_					7

ANOVA Pr > F

0.0001 0.003 0.0001 0.003 0.0001 0.0001 0.1 0.09

MANOVA Pr > F 0.0001, 0.0004 0.0001, and 0.0001.

Note 1. A pair of composites (C1,C2) is significantly different when the number corresponding to the larger composite is in the row of the smaller composite. For example, if composites 2 and 6 are significantly different for total hogs, then a 6 appears in row 2 under total hogs, but 2 does not appear in row 6.

E.6 GRAND AVERAGE RANK OVER ALL ITEMS AND ALL STATES

COMPOSITE	AVERAGE RANK	SIGNIFICANT DIFFERENCES
equal.	5.6	52736
inv.var	3.3	1465
inv.cv	3.7	145
mid.range	5.4	52736
s.inv.var	2.7**	2314637
s.inv.cv	3.8	1452
mult.frame	3.5	14

APPENDIX F

SUMMARY TABLES FOR THE MODEL INTERPRETATION OF ASB ESTIMATES

Tables F.2 through F.9 showed that the ASB's treatment of states, except for Iowa, has been the same. A comparison of the eight-state and summary results, Tables F.1 and F.10, with the individual state results, Tables F.2 through F.9, showed the dominant influence of Iowa.

Table F.11 contains the mean weighted distance between ASB second revision state models and the indicated state composite models. A state's weight was proportional to the state's mean total hogs from 1979 to 1986. Analysis of the weighted distances confirmed the dominance of Iowa on the eight-state aggregate model.

These model based interpretations of historical ASB estimates were consistent with the other analyses.

TABLE F.1
THE COMPOSITE MODEL CLOSEST TO THE INDICATED ASB MODEL EIGHT STATE TOTAL

		=====		====	====			
				asb M	ODEL			
COMPOSI'	re model	<u>1</u>	<u>2</u>	<u>3</u>	4	<u>5</u>	<u>6</u>	SUM
equal inv.va	r						*	1
inv.cv mid.ra	nge		*	*	*			3
s.inv.		*				*		2
Note 1:	The six ASI of the Desc	cripti	on of	The				
Note 2:	The asteris	sks (*) den	otes				
Note 3:	Sum gives to indicated of ASB models.	the to	tal n	umber	of t	imes	that	the
=======				====				

TABLE F.2
THE COMPOSITE MODEL CLOSEST TO THE INDICATED ASB MODEL
STATE: ILLINOIS

			ASB	MODEL			
COMPOSITE MODEL	1	2	3	4	<u>5</u>	<u>6</u>	SUM
equal inv.var inv.cv	*						1
mid.range s.inv.var s.inv.cv		*	*	*	*	*	5
Note 1: See Notes	1, 2,	and	3 for	Table	F.1.		======= ==============================

TABLE F.3
THE COMPOSITE MODEL CLOSEST TO THE INDICATED ASB MODEL
STATE: INDIANA

ASB MODEL COMPOSITE MODEL 1 2 3 4 5 6 SUM equal inv.var * * * * * * 6 inv.cv mid.range s.inv.var s.inv.cv Note 1: See Notes 1, 2, and 3 for Table F.1.	l							
<pre>COMPOSITE MODEL 1 2 3 4 5 6 SUM equal inv.var</pre>								
equal inv.var * * * * * * 6 inv.cv mid.range s.inv.var s.inv.cv				ASB I	MODEL			
inv.var * * * * * * * 6 inv.cv mid.range s.inv.var s.inv.cv	COMPOSITE MODEL	1	2	3	4	<u>5</u>	<u>6</u>	<u>sum</u>
Note 1: See Notes 1, 2, and 3 for Table F.1.	<pre>inv.var inv.cv mid.range s.inv.var</pre>	*	*	*	*	*	*	6
	Note 1: See Notes	1, 2,	and 3	for	Table	F.1.		

TABLE F.4

THE COMPOSITE MODEL CLOSEST TO THE INDICATED ASB MODEL STATE: IOWA

		=== =	=====	====	===	====	=====
			ASB M	ODEL			
COMPOSITE MODEL	1	2	3	4	<u>5</u>	<u>6</u>	SUM
equal inv.var inv.cv mid.range	*	*	*	*			4
s.inv.var s.inv.cv					*	*	2
Note 1: See Notes	1, 2,	and 3	for	 Table	F.1.		

TABLE F.5
THE COMPOSITE MODEL CLOSEST TO THE INDICATED ASB MODEL
STATE: KANSAS

		TWIE:	KAMO	no 			
			ASB	MODEL			
COMPOSITE MODEL	1	<u>2</u>	<u>3</u>	4	<u>5</u>	<u>6</u>	SUM
equal inv.var inv.cv	*			-			1
mid.range s.inv.var s.inv.cv		*	*	*	*	*	5
Note 1: See Notes	1, 2,	and 3	for	Table	F.1.		

TABLE F.6
THE COMPOSITE MODEL CLOSEST TO THE INDICATED ASB MODEL
STATE: MINNESOTA

		====	====		====		=====
			ASB M	ODEL			
COMPOSITE MODEL	1	2	<u>3</u>	4	<u>5</u>	<u>6</u>	SUM
equal inv.var inv.cv mid.range s.inv.var s.inv.cv	*	*	*	*	*	*	6
Note 1: See Notes	1, 2,	and 3	for	Table	F.1.		2222222 222222

TABLE F.7
THE COMPOSITE MODEL CLOSEST TO THE INDICATED ASB MODEL
STATE: MISSOURI

•							
====================================	=====			====	=====		======
			ASB M	ODEL			
COMPOSITE MODEL	1	2	<u>3</u>	4	<u>5</u>	<u>6</u>	SUM
equal inv.var inv.cv mid.range s.inv.var s.inv.cv	*	*	*	*	*	*	6
Note 1: See Notes 1	===== L, 2, =====	and 3	for	Table	F.1.		======

TABLE F.8
THE COMPOSITE MODEL CLOSEST TO THE INDICATED ASB MODEL
STATE: NEBRASKA

	011		~~				
		:322 = =	=====	=====	=====		
			ASB M	ODET.			
				<u> </u>			
COMPOSITE MODEL	1	2	<u>3</u>	4	<u>5</u>	<u>6</u>	SUM
equal inv.var	*						1
<pre>inv.cv mid.range s.inv.var</pre>		*	*	*	*	*	5
s.inv.cv							
Note 1: See Notes	1, 2,	and 3	for	Table	F.1.		
	=====			=====	=====	====	======

TABLE F.9
THE COMPOSITE MODEL CLOSEST TO THE INDICATED ASB MODEL
STATE: OHIO

		32222 3222	=====	.=====	=====		
			ASB M	ODEL			
COMPOSITE MODEL	1	2	<u>3</u>	4	<u>5</u>	<u>6</u>	SUM
equal inv.var inv.cv mid.range s.inv.var s.inv.cv	*	*	*	*	*	*	6
Note 1: See Notes	1, 2,	and 3	for	Table	F.1.		

TABLE F.10
THE TOTAL NUMBER OF TIMES A COMPOSITE MODEL
WAS NEAREST TO AN ASB MODEL FOR ALL 8 STATES

名城市市场的全国公司		*###		*****	34323 3	13 42 5	
			ASB M	ODEL			
COMPOSITE MODEL	1	2	<u>3</u>	4	<u>5</u>	<u>6</u>	SUM
equal inv.var inv.cv	5	2	2	2	2	2	15
mid.range s.inv.var	1 2	1 5	1 5	1	5	5	4 27
s.inv.cv	~	J	3	,	1	ĭ	2
Note 1: See Notes	1, 2,	and 3	for	Table	F.1.		

TABLE F.11
WEIGHTED MEAN DISTANCES BETWEEN THE ASB SECOND
REVISION STATE MODELS AND STATE COMPOSITE MODELS

COMPOSITE MODEL	DISTANCE TO ASB MODEL
equal	0.203
inv.var	0.203
inv.cv	0.166 ***
mid.range	0.201
s.inv.var	0.193
s.inv.cv	0.171

Note 1: The weights are proportional to the state totals.

Note 2: The *** symbol shows the composite model with the smallest distance to the ASB estimates and revisions.